A STUDY OF RADIATIVE MAGNETIC SHOCKS WITH THE ASSUMPTION OF ARTIFICIAL VISCOSITY

THESIS PRESENTED

BY

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TO FULFIL PARTIAL REQUIREMENT FOR

THE DEGREE OF MASTER OF PHILOSOPHY OF

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JHANSI

(INDIA)

Department of
Mathematics & Statistics

Supervisor
Dr. V. K. Singh

1989

DEDICATED

TO MY GRAND FATHERS

LATE SHRI P. S. AGARWAL

and

LATE SHRI R. K. GUPTA.

CERTIFICATE

****

THIS IS TO CERTIFY that the wrok embodied in the THESIS entitled " A STUDY OF RADIATIVE-MAGNETIC SHOCKS WITH THE ASSUMPTION OF ARTIFICIAL VISCOSITY " being submitted by SURABHI AGARWAL to fulfil the partial requirement for the Degree of M.Phil. of Bundelkhand University, Jhansi (U.P.) is up-to the mark both in its academic contents and in the quality of presentation.

I further certify that this work has been done by her under my supervision and guidance.

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Dated, Jhansi
December 20 ,1989

PREFACE

****

The Present work is an out-come of the studies made by me during the course of studies for " M.Phil." at the Department of Mathematics and Statistics, Bundelkhand University, Jhansi.

This THESIS consists of three Chapters numbered I, II and III; every Chapter is divided into several sections (progressively numbered like 1.1, 1.2, 2.1, 3.1------------). The formula and equations are numbered progressively within every section.

Chapter I has been divided into several parts, first is History of Fluid Variables and others are the Fundamental Equations described in detail. The existence of surface of discontinuity and other properties of shock wave is detailed out. The generalised Rankine-Hugoniot Jump relation is derived in the preceding section. Similarity principle is defined such as the partial differential equation reduces to ordinary differential equation with independent variable. The structure of a radiative shock wave in a viscous compressible thin layer adjacent to the surface of body has been studied and expressions for the variation of the physical quantities across the shock have been derived.

The present work is with well-known result notation given at the beginning of the Thesis. References are given at the end of the Thesis and are denoted in Thesis of numbers

in square brackets.

The two papers are solved in Chapter II and Chapter III such as ' Spherical Shock Wave in Viscous Magnetogas-dynamics ' and ' Self-Similar Cylinderical Shock Wave with Radiation Heat Flux ' respectively.

I have unique privilege of working under the able supervision of **Dr. V.K. SINGH**, Department of Mathematics & Statistics, Bundelkhand University, Jhansi (U.P.). I started my work under his distinguished guidance during Session 1988-89. I am highly grateful to Dr. V.K.SINGH for his continuous encouragement and suggestions.

I am very much indebted to **Dr. P.N.SHRIVASTAVA**, Head of the Department of Mathematics & Statistics, Bundelkhand University, Jhansi (U.P.), for his continuous encouragement and sincere advice in preparing my Thesis. I would fail in my duty if I do not thank **Dr. V.K.SEHGAL**, Department of Mathematics & Statistics, Bundelkhand University, Jhansi (U.P.), for his valuable suggestions.

My special thanks are to **Dr. V.D.SHARMA**, Department of Mathematics, Professor I.I.T., Bombay for his good wishes and to my Uncle **Dr. M.M. GUPTA**, Department of Mathematics, Professor, George Washington University, Washington (U.S.A.) for his distinguished appreciation of my work.

It would be failing in duties if I forget to acknow-ledge most sincere contribution of my parents **Mr. J.S.AGARWAL** & **Smt. PUSHPA AGARWAL** who have always been a constant source of inspiration in my life. I am also grateful to my brothers

SAURABH & SAMEER for their valuable assistance and active co-operation.

In conclusion I should like to thank my friend KUMARI NEETA AGARWAL (also a M.Phil. student) and Mr. K.K.SHRIVASTAVA, (Research Scholar) for their co-operation and help in completing my work.

Surabhi Agarwal

(SURABHI AGARWAL)

Dated, Jhansi
December 20, 1989

C O N T E N T S

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The following two papers have been in Chapter II and Chapter III.

Spherical Shock Waves in viscous magnetogasdynamics

B.G.Verma, R.C.Srivastava, and V.K.Singh
Department of Mathematics, University of Gorakhpur, U.P. (India).

Astrophysics and Space Science 111(1985) 253-263
0004-640X/85.15

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Self-Similar Cylindrical Shock Waves with Radiation Heat Flux

J.B.Singh
Department of Mathematics, S.G.R.Post Graduate College, Dobhi, Jaunpur, U.P. (India)

Astrophysics and Space Science 102(1984) 263-268,
0004-640X/84/1022-0263500, 90

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LIST OF IMPORTANT NOTATIONS

ρ	.....	Density of fluid particle
p	.....	Pressure of fluid particle
T	.....	Temperature of fluid particle
$\bar{q}$	.....	Velocity of fluid particle
R	.....	Gas constant
e	.....	Internal energy of fluid
$\bar{H}$	.....	Magnetic field strength
$\bar{J}$	.....	Electric Current Density
$\bar{E}$	.....	Electric field strength
ϵ	.....	Dielectric constant
μ	.....	Constant of Permitivity
$\bar{B}$	.....	Magnetic induction vector
σ	.....	Current density
$\bar{D}$	.....	Electric displacement density
c	.....	Velocity of light
ν	.....	Coefficient of viscosity
a	.....	Speed of sound
M	.....	Mech. number
M_A	.....	Alfven's Mech number
F	.....	Force
r	.....	Radial distance
γ	.....	Adiabatic gas index
N	.....	Non dimensional radiation parameter

CHAPTER-I

INTRODUCTION

Fluid Dynamics is that branch of Science which is concerned with the study of motion of fluid or that of bodies in contact with liquids and gases. Which are classified as fluid. We regard liquid as a incompressible fluids for all practical purpose and gases as compressible fluid. In general, fluids have five physical variable; Density, Velocity, temprature, pressure and velocity [1] [2]

Since the phenomenom considered in fluid dynamics are macroscopic, a fluid is regarded as a continuous medium, this means that any small volume element in the fluid is always supposed so large that it still contain a many number of moleculars so when we take of infinitely small element of volume. We always mean those which are physically small it is found to consist of molecules in random motion and separated from one another by distance which are at least comprable with molecular size. In the case of gases, the separation distance are great; in the case of liquid, they are less great and in the case of solids even less so.

The mathematical description of state of a moving fluid is affected by means of function which gives the distribution of fluid velocity $V=v(x,y,z,t)$ and of any two thermodynamics quantities pertaining to the fluid. For example the pressure $P(x,y,z,t)$ and the density $\rho(x,y,z,t)$ All the thermodynamics quantities are determined by the values of any two of them together with equation of state.

Hence if we are gives five quantities namely the three components of velocity v, the pressure p, and the density ρ. The state of moving fluid is completely determined [2] [3]

There are two method for finding fluid motion mathematically, these are " LAGRANGIAN " and " EULERIAN " method for refer to " INDIVIDUAL TIME RATE OF CHANGE " and " LOCAL TIME RATE OF CHANGE " respectively.

They are as follows:-

(1) "INDIVIDUAL TIME RATE OF CHANGE"

In this method we study the history of each particle i.e any fluid particle is studied and is pursued on its onward course. Observing the change in velocity, pressure, and density at each point and at each instant.

The fundamental equations of motion in Lagrangian form are non-linear and hence it leads to many difficulties while solving a problem. In fact, it is employed with an advantage only in some one-dimensional (involving one space coordinate) problem [?] [5]

(2) "LOCAL TIME OF RATE OF CHANGE"

In this method we study a general point in space occupied by fluid and study the whatever changes take place in velocity, pressure, and density as the fluid posses through this point rather than the variation of velocities, pressure and acceleration etc.

As they follows their own path. In it the individual fluid particles are not identified instead a point in chosen and changes in velocity etc. are studied as the fluid passes through the chosen fixed point. [5]

We can drive the 'LOCAL' and 'INDIVIDUAL' time rates as following or relationship between Lagrangian and Eulerian method.

Let the fluid motion be associated by a scaler function $\phi(x, y, z, t)$ or $\phi(r, t)$ keeping the point P (x, y, z) fixed. The change in during an interval of time 'δt' is

$$\phi(xyz, t+\delta t) - \phi(xyz, t)$$

or

$$\phi(\vec{r}, t+\delta t) - \phi(\vec{r}, t)$$

Hence the'local time rate of changes' $\frac{\partial \phi}{\partial t}$ is given by

$$\frac{\partial \phi}{\partial t} = \lim_{\delta t \to 0} \frac{\phi(\vec{r}, t+\delta t) - \phi(\vec{r}, t)}{\delta t}$$

Now keeping the particle fixed the change in ϕ is

$$\phi(\vec{r}+\delta \vec{r}, t+\delta t) - \phi(\vec{r}, t)$$

When $\delta \vec{r}$ is the change in position of fixed particle during the short time δt, therefore,

$$\frac{d\phi}{dt} = \lim_{\delta t \to 0} \frac{\phi(\vec{r}+\delta \vec{r}, t+\delta t) - \phi(\vec{r}, t)}{\delta t}$$

Given the " individual time rate of change "

Let q (u, v, w) be the velocity of fluid particle, such that

$$\vec{q} = u\hat{i} + v\hat{j} + w\hat{k}$$

and $$\frac{dx}{dt} = u.$$ etc.

then;

$$\frac{d\phi}{dt} = \frac{\partial\phi}{\partial t} + \frac{\partial\phi}{\partial x}\frac{dx}{dt} + \frac{\partial\phi}{\partial y}\frac{dy}{dt} + \frac{\partial\phi}{\partial z}\frac{dz}{dt}$$

$$\frac{d\phi}{dt} = \frac{\partial\phi}{\partial t} + u\frac{\partial\phi}{\partial x} + v\frac{\partial\phi}{\partial y} + w\frac{\partial\phi}{\partial z}$$

$$\frac{d\phi}{dt} = \frac{\partial\phi}{\partial t} + (u\hat{i} + v\hat{j} + w\hat{k})(\hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z})$$

$$\frac{d\phi}{dt} = \frac{\partial\phi}{\partial t} + (\nabla\phi)\cdot q$$

$$\frac{d\phi}{dt} = \frac{\partial\phi}{\partial t} + q\cdot\nabla q.$$

This is relation between the two time rates.

FUNDAMENTAL EQUATIONS GOVERNING THE FLOW OF GAS

We consider the fluid flow, we wish to find the velocity distribution as well as the states of the fluid over all spaces for all time. If the fluid is a single gas ordinarily a knowledge of the three velocity components (u,v,w) the temperature T, the pressure P, and the density ρ of the fluid which are function of spatial coordinate and time is desired. Hence we study three dimensional flow and consider all three components of velocity together with the pressure, density and temprature of a fluid as a function of three spatial coordinates x,y and z and the time t.

We shall consider only inviscid and non heat conducting fluid because the effects of viscosity and heat conduction are usually negligible except near a solid boundary or inside a shock. [2]

(a) EQUATION OF STATE

Which connected the temperature T, the pressure P and the density ρ of the fluid, for a perfact gas, this equation may be written as

$$p = \rho R T \qquad [1.1]$$

R being a gas constant.

(b) EQUATION OF CONTINUITY

Which express the conservation of mass of the fluid. If q represent the velocity of the fluid at any time t, the

equation of countinuity may be written as

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \vec{q}) = 0 \qquad [1.2]$$

(c) EQUATION OF MOTION

Which express the relation of conservation of momentum in the fluid. Neglecting the body force and considering only the inertial forces and pressure forces, the equation of motion may be written as [2]

$$\frac{D\vec{q}}{Dt} = - \operatorname{grad} p$$

Where $\frac{D}{Dt}$ is the usual mobile operator given by

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + (\vec{q} \cdot \operatorname{grad}) \qquad [1.3]$$

(d) EQUATION OF ENERGY

Which express the conservation of energy in the fluid. This can be expressed as [1] [2]

$$\frac{De}{Dt} + \rho \frac{D}{Dt}\left(\frac{1}{\rho}\right) = Q \qquad [1.4]$$

Where e and Q are the internal energy and the energy generated by external sources per unit time per unit mass of the fluid.

(e) MAGNETOHYDRO DYNAMICS EQUATION

Under magnetohydro dynamics we study the motion of electrically conducting fluids in the presence of electromagnetic fields. The equation of motion of fluid element from fluid mechanics with the macroscopic phenomena logical

maxwell equation from electro dynamics and the thermal ~~and~~ equation of state from thermodynamics. The electric current is induced in a conductor moving in a magnetic field. These currents being in a magnetic field suffer a mechanical force called Lorentz force.

Magnetohydro Dynamics its peculier interest and difficulty to this interaction between the field and the motion.

Neglecting maxwell's displacement currents and taking magnetic permeability μ as unity. The field equation are

$$\mathrm{Curl}\ H = 4\pi J + \epsilon\,(\partial \vec{E}/\partial t)$$

$$\mathrm{Curl}\ E = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t}$$

$$\mathrm{div}\ \vec{E} = \frac{4\pi}{\epsilon}\vec{q}$$

$$\mathrm{div}\ \vec{H} = 0 \qquad [1.5]$$

Where the symbols $\vec{H}$, $\vec{J}$, $\vec{q}$, $\vec{E}$ and c denote the magnetic field, current density, electric intensity and velocity of light respectively, ϵ is dielectric constant and $\vec{q}$ the charge density. In addition the constitutive equation are

$$J = \sigma\vec{E}, \quad \vec{B} = \mu\vec{H} \quad \text{and} \quad \vec{D} = \epsilon\vec{E} \qquad [1.6]$$

" ORIGIN OF DISCONTINUITY "

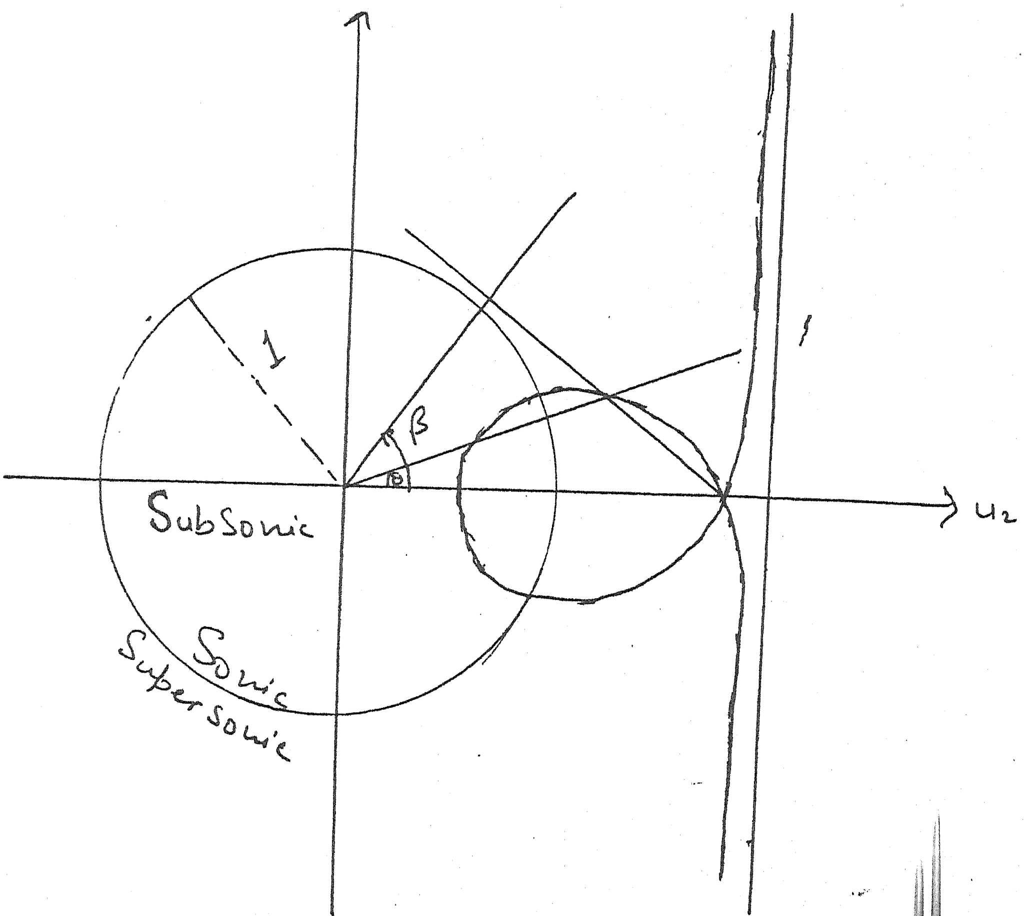
The laws of conservation of mass, momentum and energy that from the basis for the equation of inviscid flow of a non-conducting to not necessarily assume continuity of the flow variables. These laws are originally formulated in the form of differential equations. Simply because it is assumed at the beginning that the flow is continious. These laws, however can also be applied to these flow regions where the variable undergo a discontinious change. From the mathematical point of view, a discontinuity can be regarded as the limiting case of very large but finite gradients in the flow variables across a layer whose thickness tends to zero.

Since the dynamics of an inviscid and non-conducting gas there are no characteristic lengths, the possibility of the existance of arbitrary thin transition layers is not excluded. In the limit of vanishing thickness, these layers reduce to discontinuity such discontinuity are called SHOCK WAVE.

Actually the shock wave is not a simply surface of discontinuity but a very narrow region in which a large variation of pressure and velocity occurs it is possible to estimate the thickness of a shock wave approximately. From this approximate estimate we may show that the extent of the shock thickness is very small and therefore neglected and may be considered as a surface of discontinuity. [1] [2] [4] [6]

Shock wave appear in many compressible flow problem often a shock wave with heat addition is the "CONDENSATION SHOCK"

In the actual flow of air, there is normally a certain amount of moisture present which is a vapour state. Second one is heat addition in "DETONATION WAVE" which arises from the rapid transformation of explosive material and the flow direction is not perpendicular to the shock wave front such in the case of an "OBLIQUE SHOCK". There is a simple graphical relation for the velocity components across an oblique shock known as the shock Polar.



THE MATHEMATICAL TREATMENT OF SHOCK WAVE:

We can define a density $\rho(x,t)$ per unit length and a flux $q(x,t)$ per unit time. We can define a flow velocity $v(x,t)$ by

$$v = q/\rho$$

We can stipulate that the rate of change of the total amount of it in any section $x_1 > x > x_2$ must be balanced by the net inflow across x_1 and x_2. That is

$$\frac{d}{dx}\int_{x_2}^{x_1} \rho(x,t)\,dx + q(x_1,t) - q(x_2,t) = 0$$

If $\rho(x,t)$ has continious derivatives, we may take the limit as $x_1 \longrightarrow x_2$ and obtain the conservation equation.

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = 0$$

The simplest wave problems arise when it is reasonable on either theoritical or empirical grounds, to postulate a functional relation between q and ρ.

$$q = Q(\rho)$$

Consider first the mathematical question of whether discontinuities are possible. Suppose there is a discontinui at x = S (t) and that x_1 and x_2 are chosen so that ~~$x_1 > S(t) >$~~ $x_1 > S(t) > x_2$ suppose ρ and q and their first derivatives are continious in $x_1 \geq x > S(t)$ and in $S(t) > x \geq x_2$

Finite limit as $x \longrightarrow s(t)$.

$$q(x_2,t) - q(x_1,t) = \frac{d}{dt}\int_{x_2}^{s(t)} \rho(x,t)\,dx.$$

$$+ \frac{d}{dt}\int_{s(t)}^{x_1} \rho(x,t)\,dx.$$

We consider the differential equation we have infinit number of conservation laws

$$u_t + u u_x = 0$$

$$\frac{\partial}{\partial t} u + \frac{\partial}{\partial x}\left(\frac{u^2}{2}\right) = 0$$

$$\frac{\partial}{\partial t}(u^2) + \frac{\partial}{\partial x}\left(\frac{2}{3}u^3\right) = 0$$

$$\vdots$$

$$\frac{\partial}{\partial t}(u^n) + \frac{\partial}{\partial x}\left(\frac{n}{n+1}u^{n+1}\right) = 0$$

integrating it

$$\int_{x_1}^{x_2} \frac{\partial}{\partial t} u^n\,dx = \frac{n}{n+1}\left\{u^{n+1}(x_1) - u^{n+1}(x_2)\right\}$$

we have

$$\int_{x_1}^{x_2} \frac{\partial u^n}{\partial t}\,dx = \int_{x_1}^{X(t)} \frac{\partial u^n}{\partial t}\,dx + \int_{X(t)}^{x_2} \frac{\partial u^n}{\partial t}\,dx.$$

$$\int_{x_1}^{X(t)} \frac{\partial u^n}{\partial t}\, dx = \int_{x_1}^{X_1} \frac{\partial u_n}{\partial t} + u^n(\overline{X}(t), t)\frac{dx}{dt}$$

$$\int_{x_1}^{X(t)} \frac{\partial u^n}{\partial t} dx = \left[\frac{\partial}{\partial t}\int_{x_1}^{X(t)} u^n dx + u^n(\overline{x}, t)\frac{dx}{dt}\right]$$

$$+ \frac{\partial}{\partial t}\int_{X(t)}^{x_2} u^n dx - u^n(x^+, t)\frac{dx}{dt}\Big]$$

$$\int_{x_1}^{x_2} \frac{\partial u^n}{\partial t} dx = \frac{\partial}{\partial t}\int_{x_1}^{x_2} u^n dx + s(u_\ell^n - u_R^n)$$

$$\int_{x_1}^{x_2} \frac{\partial u^n}{\partial t} dx = \frac{\partial}{\partial t}\int_{x_1}^{x_2} u^n dx + s'(u_\ell^n - u_R^n)$$

$$\int_{x_1}^{x_2} \frac{\partial u^n}{\partial t} dx = \frac{n}{n+1}\left[u^{n+1}(x_1) - u^{n+1}(x_2)\right]$$

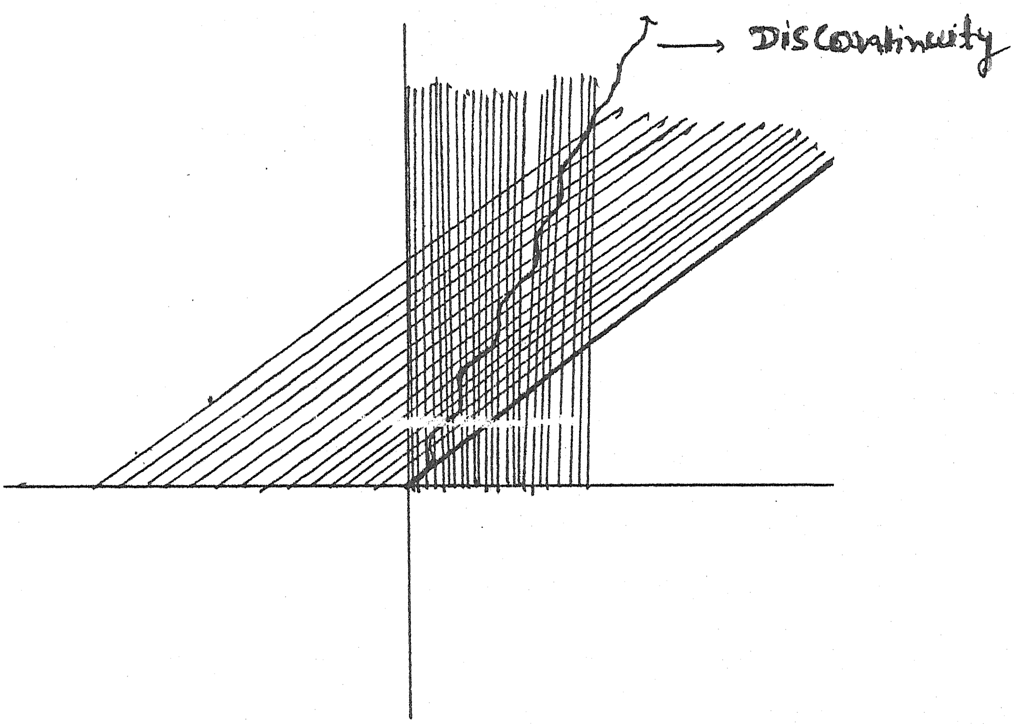
$$x_1 \longrightarrow x^-$$
$$x_2 \longrightarrow x^+$$

$$s'[u^n] = \frac{n}{n+1}\left[u_\ell^{n+1} - u_R^{n+1}\right]$$

$$s'[u^n] + \frac{n}{n+1}\left[u^{n+1}\right] = 0$$

S is speed of propogate the jump condition

$$S' = \frac{n}{n+1} \frac{[u^{n+1}]}{[u^{n}]}$$



They will show that weak solution of a conservation law.

It is the true conservation equation then it may be deduced as the shock condition by the same argument. That correct choice of weak solution is made on the basis of which quantities are really conserved across the shock.

TYPES OF OBLIQUE SHOCK WAVE

(1) Plane Shock

(2) Cylinderical Shock

(3) Spherical Shock

One of the interesting case of unstudy an isentropic flow in the decay of a strong shock we may consider such a problem as depending on time t and on a single spatial coordinates, i.e. we may consider either a plane (or normal), cylindrical, or spherical shock wave [4] [6]

(1) PLANE SHOCK

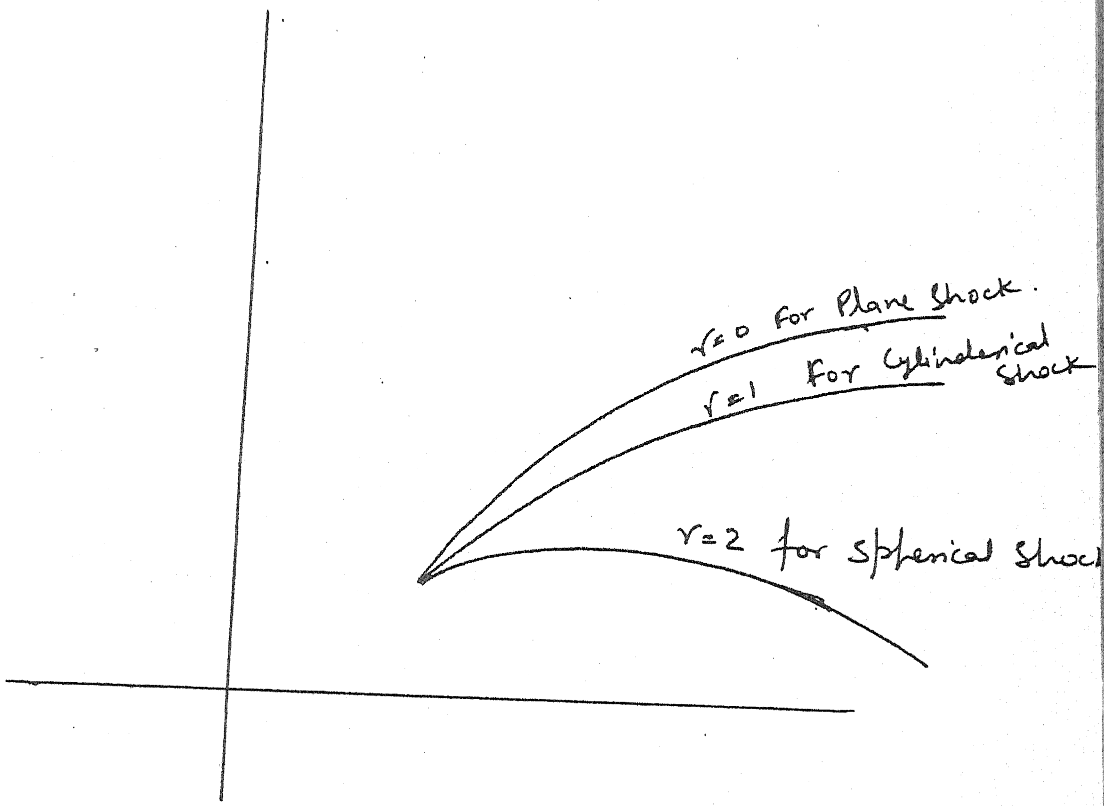
Consider a normal shock to be a surface of discontinuity in velocity, pressure, density and temperature of the fluid. We shall find the relation between these quantities in front of and behind the shock, we choose the coordinate system such that the shock is stationery and the fluid moves it. Under these conditions the flow is steady, we assume that the fluid is an ideal gas, so that the perfect gas law holds both infront and behind the shock, and the specific heat are constant, we assume that the velocities of fluid particle are perpendicular to the shock front.

(2) CYLINDRICAL SHOCK

If there is an instantaneous release of energy along a line. The shock front such developed are known as cylindrical shock wave.

(3) SPHERICAL SHOCK

There is an instantaneous release of energy along a point. The shock front such developed are known as spherical shock wave.



LAW OF MOTION OF GAS PARTICLE

A shock wave may be produced by an instantaneous energy release, such as the explosion of an atomic bomb. We assume that a finite amount of energy is suddenly released in an infinitely concentrated form in the spherical case, and as energy per unit length E in the cylindrical case. If the value of energy release E are the same for both cases.

That the air in an ideal gas so that specific heats are constant.

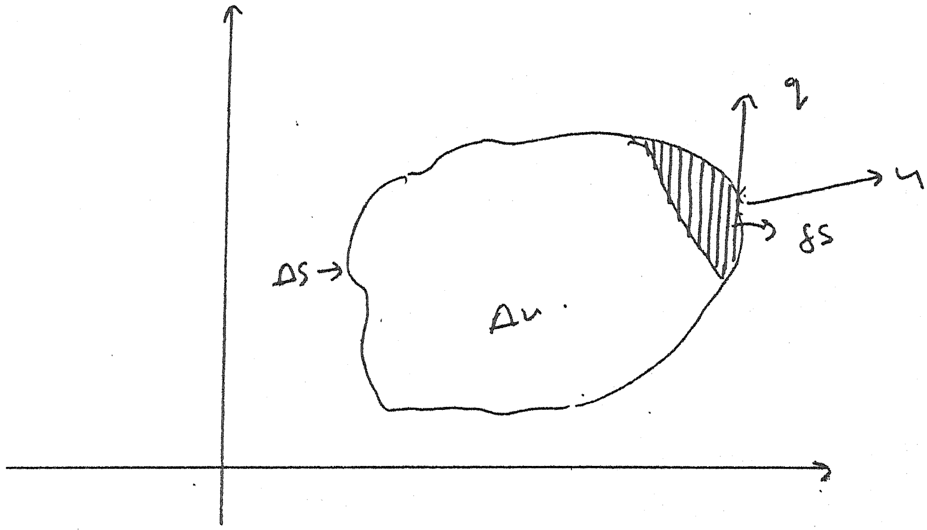
The fundamental equation are

$$
\left.
\begin{aligned}
&\frac{Du}{Dt} = -\frac{1}{\rho}\,\frac{\partial p}{\partial r} \\
&\frac{D\rho}{Dt} + \rho\left(\frac{\partial u}{\partial r} + \delta\,\frac{u}{r}\right) = 0 \\
&\frac{D}{Dt}\left(\frac{p}{\rho^{\gamma}}\right) = 0 \\
&\text{or} \qquad \frac{DS}{Dt} = 0
\end{aligned}
\right\} \qquad [1.7]
$$

We discuss here the fundamental equation in detail which gives the basic inpastructure to the problem elaborated in the later part of the thesis.

(1) EQUATION FOR THE LAW OF CONSERVATION OF MASS

When a region of a fluid contains neither sources nor sinks that is to say when there are no inlets or outlets through which fluid can enter or leave the region. The amount of fluid within the region in conserved in accordance with the principle of conservation of matter [1]



Let ΔS be a closed surface drawn in the fluid and taken fixed in space and let $\rho = \rho\,(x, y, z, t)$ be the fluid density (mass per unit volume) at any point (x, y, z) of the fluid in Δv at any time t. Suppose n is the unit out ward drawn normas at any surface element δS of ΔS where $\delta S < \Delta S$, then if q is fluid velocity at the element δS. The normal component of q measured out ward from Δv is n.q. Thus

Rate of effiux of fluid mass per unit time

across $\delta S = \rho n \cdot q\, \delta S$

Total rate of mass flow out of Δv across

$$\Delta s = \int_{\Delta s} \rho n \cdot q \, ds$$

Total rate of mass flow into Δv.

$$= -\int_{\Delta s} n \cdot (\rho q) \, ds$$

$$= -\int_{\Delta v} \nabla (\rho q) \, dv$$

At time t, the mass of fluid with in the element is $\int_{\Delta v} \rho \, dv$

Local rate of mass increase within Δv

$$= \frac{\partial}{\partial t} \int_{\Delta v} \rho \, dv.$$

$$= \int_{\Delta v} \frac{\partial \rho}{\partial t} \, dv.$$

in the absence of sources and sinks Δv, matter is not created or destroyed in this region so that

$$\int_{\Delta v} \frac{\partial \rho}{\partial t} \, dv = -\int_{\Delta v} \nabla \cdot (\rho q) \, dv.$$

$$\text{or,} \quad \int_{\Delta v} \left[\frac{\partial \rho}{\partial t} + \nabla (\rho q) \right] dv = 0$$

for all volume Δv if

$$\frac{\partial \rho}{\partial t} + \nabla (\rho q) = 0 \qquad [1 \cdot 8]$$

Equation [1.8] is the general equation of continuity which must always holds at any point of a fluid free from sources and sinks. Since

$$\nabla(\rho q) = \rho \nabla q + q \nabla \rho$$

Substitute the value of $\nabla(\rho q)$ in equation (1.8) then

$$\frac{\partial \rho}{\partial t} + \rho \cdot \nabla q + q \cdot \nabla \rho = 0$$

$$\frac{d\rho}{dt} + \rho \nabla q = 0 \qquad [1.9]$$

$$\frac{d}{dt}(\log \rho) + \nabla \cdot q = 0 \qquad [1.10]$$

In second and third equation d/dt denots differentiation following the fluid motion and the operational equivalence

$$\frac{d}{dt} = \frac{\partial}{\partial t} + q \cdot \nabla$$

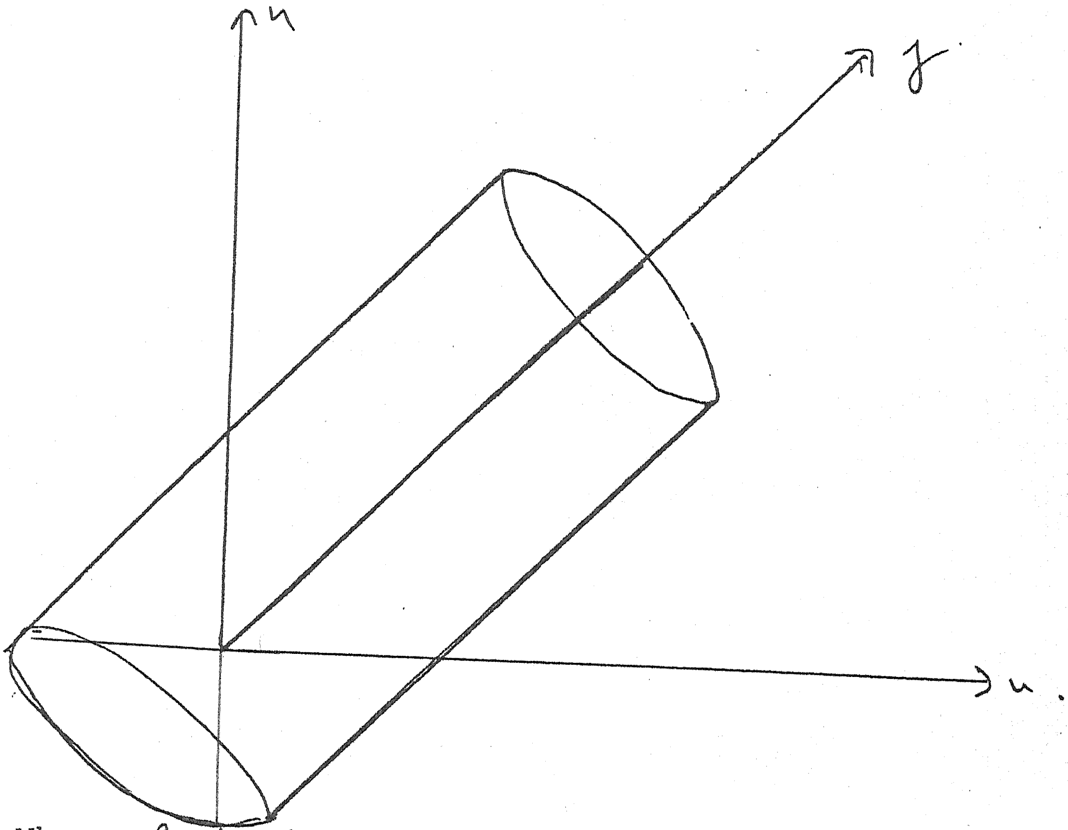
has been used.

(2) .THE EQUATION OF MOTION

When the fluid moves an electric and magnetic field the body force F per unit volume consiste of three parts. Gravitational, Electric and Magnetic, the gravitational body force per unit volume is ρg where g is the acceleration due to gravity. An elemental volume δv of the fluid would contain a charge of amount $q\,\delta v$ so that the force on it due to an electric field by intensity E would

be $(q\,\delta v)\,E$. 2

The normal cross section of a fluid element whose length δs is the direction of j. This element moves along with the local fluid velocity v in a magnetic field of intensity H.



Where ρ is the fluid density, v the fluid velocity. F the body force per unit volume, p the fluid pressure, ν kinetic coefficient of viscosity

for ordinary viscous fluid

$$\rho \frac{D\vec{u}}{Dt} = F - \operatorname{grad} p + \nu \nabla^2 \vec{v}$$

If the fluid is under the effect of magnetic field

$$\rho \frac{Du}{Dt} = J \times B - \operatorname{grad} p + \nu \nabla^2 \vec{v}.$$

When external force $f^{ex.}$ & mechanical force $f^{n.}$ has been neglected. $B = \mu \vec{H}$.

$$\rho \frac{D\vec{v}}{Dt} = (\mu \vec{J} \times \vec{H}) - \operatorname{grad} p + \nu \nabla^2 \vec{v}$$

As $$\frac{1}{4\pi} \operatorname{Curl} H = J + \frac{\partial D}{\partial t} \frac{1}{4\pi}.$$

neglecting the displacement current

$$\frac{1}{4\pi} \operatorname{Curl} H = J$$

Substitute it

$$\rho \frac{Dv}{Dt} = \left(\frac{\mu}{4\pi} \operatorname{Curl} \vec{H} \times \vec{H}\right) - \operatorname{grad} p + \nu \nabla \vec{v}$$

by vector identity

$$\begin{cases} \operatorname{grad} \frac{H^2}{2} = (\vec{H} \times \operatorname{Curl} \vec{H}) + (\vec{H} \nabla) \vec{H} \\ \frac{\mu}{4\pi} (\operatorname{Curl} \vec{H} \times \vec{H}) = -\operatorname{grad}\left(\frac{\mu H^2}{8\pi}\right) + \frac{\mu}{4\pi} (H \nabla) \vec{H} \end{cases}$$

Substitute it

$$\rho \frac{Du}{Dt} = -\operatorname{grad} \frac{\mu H^2}{8\pi} + \frac{\mu}{4\pi} (\vec{H} \nabla) H - \operatorname{grad} p + \nu \nabla^2 \vec{v}$$

$$\rho \frac{Du}{Dt} = -\operatorname{grad}\left(p + \frac{\mu H^2}{8\pi}\right) + \frac{\mu}{4\pi} (\vec{H} \nabla) \vec{H} + \nu \nabla^2 \vec{v}$$

Where $p^* = \left(p + \frac{\mu H^2}{8\pi}\right)$ is magnetic pressure.

$$\rho \frac{Du}{Dt} = -\operatorname{grad} p^* + \frac{\mu}{4\pi} (\vec{H} \nabla) \vec{H} + \nu \nabla^2 \vec{H}$$

ν is coefficient of viscosity

This gives the equation of motion for viscous conducting fluid.

(3) EQUATION OF ENERGY

For one dimentional steady flow of an inviscid

compressible fluid in a channel, the momentum law gives [1] [2]

$$\frac{dp}{\rho} + q\,dq = 0$$

Where q is the velocity, p is the pressure and ρ is the density of the fluid.

Since we consider only inviscid and non heat conducting fluid, the conservation of energy gives the energy equation as

$$dQ = Tds = dE + p\,d(1/\rho)$$

$$dQ = dH - \frac{1}{\rho}\,dP \qquad - [1.11]$$

which, in vector notation, becomes

$$T\nabla S = \nabla H - \frac{1}{\rho}\nabla p$$

it is sometimes convenient to introduce a stagnation enthalpy H_0 defined as

$$H_0 = H + \frac{1}{2}q^2 \qquad [1.12]$$

This represents the sum of heat energy H and kinetic energy of fluid per unit mass, A knowledge of the time rate of change of the stagnation enthalpy may be gottam by the following procedure.

From equation [1.12] we have

$$\frac{DH_0}{Dt} = \frac{DH}{Dt} + \frac{D}{Dt}\left(\frac{1}{2}q^2\right)$$

$$\frac{DH_0}{Dt} = \frac{DH}{Dt} + \vec{q}\,\frac{Dq}{Dt}$$

The energy equation [1.11] may be written as

$$T \frac{DS}{Dt} = \frac{DH}{Dt} - \frac{1}{\rho} \frac{D\rho}{Dt}$$

$$T \frac{Ds}{Dt} = \frac{DH}{Dt} - \frac{1}{\rho} \left[\frac{\partial \rho}{\partial t} + (\vec{q} \cdot \nabla) \rho \right]$$

Combinig equation [1.11] and [1.12] gives

$$\frac{DH_c}{Dt} = T \frac{Ds}{Dt} + \frac{1}{\rho} \frac{\partial \rho}{\partial t} + q \left\{ \frac{Dq}{Dt} + \frac{1}{\rho} \nabla p \right\}$$

$$\frac{DH_c}{Dt} = T \frac{Ds}{Dt} + \frac{1}{\rho} \frac{\partial p}{\partial t}$$

$$\frac{DH_c}{Dt} = \frac{DQ}{Dt} + \frac{1}{\rho} \frac{\partial P}{\partial t}$$

Which shows the time rate of change of stagnation enthalpy with heat addition. For adiabatic flow, the heat added is zero, i.e. $dQ = 0$ and we have

$$\frac{DH_0}{Dt} = \frac{\partial H_0}{\partial t} + (q \cdot \nabla) H_0$$

$$\frac{DH_0}{Dt} = \frac{1}{\rho} \frac{\partial P}{\partial t} \qquad [1.13]$$

and

$$\frac{DS}{Dt} = \frac{\partial S}{\partial t} + (q \cdot \nabla) S = 0 \qquad [1.14]$$

Equation [1.13] and [1.14] are the energy equation for the three dimensional adiabatic flow of comprassible flow.

EQUATION OF STATE :

The measurable quantities of a compressible substance are its pressure p, density ρ, and temperature T. It is found that these quantities are connected through a functional relation of the form.

$$F(p, \rho, T) = 0$$

Where F in a single valued function of the variables p, ρ, T such an equation is known as the equation of state of the substance. The form assumed by F depends on the nature of the substance.

That the pressure in a fluid is ascribable to the momentum change produced in its molecules on impinging against a small plane rigid surface. The density is proprtional to the number of molecular structure of matter. In addition, since the randomly moving molecules of a substance have a mean kinetic energy associated with them, this leads to the notion of temperature, being defined as a measure of this kinetic energy of random molecular motion.

For the moment we confine our attention to gaseous substances. In some cases, the molecules of a gas have but negligible volume and there are virtually no mutual attractions between the individual molecules. Such a gas is said to be a perfect gas and its equation of state assume the simple form.

$$p = R \rho T$$

Where R is a constant for the particular gas under consideration.

Let be some thermodynamic is function of a substance expressible in terms of any two of the measurable quantities p, ρ T. We have seen that this is always possible whenever the equation of state of the substance is known, it may happen that if we express as a functioning, say, p and v (v = 1/ρ), then in changing from an initial state A to another state B the value of depends only on the conditions at A and B and not at all on the paths joining them. In these circumstances, Φ is termed a function of state.

Suppose that the differential of Φ is expressible in terms of dp and dv by the relation.

$$d\Phi = M(p,v)\,dp + N(p,v)\,dv .$$

If $\Phi_B - \Phi_A$ is independent of the path taken in changing from the conditions at A to those at B, then $d\Phi$ must be an exact differential. The necessary and sufficient condition for this is

$$\left(\frac{\partial M}{\partial v}\right)_p = \left(\frac{\partial N}{\partial p}\right)_v .$$

then Φ (p,v) is a function of state

The first law of thermodynamics gives

$$dQ = p\,dv + dU$$

for a perfect gas, $dU = C_v dT$

$$dU = (C_v/R)\,d(pv)$$

using the equation $pv = RT$

Thus for such a gas

$$dQ = \left\{1 + \left(\frac{C_v}{R}\right)\right\} p\,dv + \left(\frac{C_v}{R}\right) v\,dp$$

$$dQ = (C_p\, p\, dv + C_v\, v\, dp)/R$$

identity dQ with $d\phi$, we have

$$M = \left(\frac{C_v}{R}\right)_v \quad , \quad N = \left(\frac{C_p}{R}\right)_p$$

Now

$$\left(\frac{\partial M}{\partial v}\right)_p = \frac{C_v}{R} \quad ; \quad \left(\frac{\partial N}{\partial p}\right)_v = \frac{C_p}{v}$$

Since $C_p \neq C_v$, dQ is not an exact differential. This means that the quantity of heat Q added to a unit mass of gas is not a function of state, it depends on the both chosen in the change from a given state A to a second state B.

Using pv = RT the relation for dQ for a perfect gas may be written in the alternative form

$$dS = \left(\frac{C_p}{v}\right)dv + \left(\frac{C_v}{p}\right)dp$$

where $ds = dQ/T$

$$\frac{\partial}{\partial p}\left(\frac{C_p}{v}\right) = 0 \quad ; \quad \frac{\partial}{\partial v}\left(\frac{C_v}{p}\right) = 0$$

so that

$$\frac{\partial}{\partial p}\left(\frac{C_p}{v}\right) = \frac{\partial}{\partial v}\left(\frac{C_v}{p}\right)$$

This shows that dS is an exact differential. We may integrate this differential to give

$$S - S_0 = C_p \log v + C_v \log p,$$

or,
$$\log (p v^{\gamma}) = (S - S_0)/C_v$$

thus

$$p v^{\gamma} = \exp \{(S - S_0)/C_v\}$$

The quantity S is called the entropy per unit mass, dS is the entropy differential. Flour for which S is constant and called isentropic

$$p v^{\gamma} = \text{Constant}$$

If a change is made so that the entropy of every single particle of the working substance stays constant, then such a change is termed isentropic when the entropy of every single quantity of a substance of fixed mass is the same and stays constant in any change, the change is said to be homentropic.

(4) EQUATION OF MAGNETIC FIELD

Consider a non conducting compressible fluid could transmit sound waves which are logitudenal in kind, no form of transverse propogation in possible for such a fluid. More over an incompressible fluid, which a non conducting can not support any kind of wave motion at all. When the fluid is conducting, however transverse wave motion through it is possible when ever a magnetic field is present [1] [2]

$$\text{Curl } H = 4\pi J + \frac{\partial D}{\partial t}$$

neglecting displacement density current

$$J = \sigma \left[\vec{E} + \mu (\vec{V} \times \vec{H}) \right]$$

4π multiply both side

$$4\pi J = 4\pi\sigma \left[E + \mu (\vec{V} \times \vec{H}) \right]$$

$$\text{Curl } H = 4\pi\sigma \left[E + \mu (\vec{V} \times \vec{H}) \right]$$

Taking curl in both side

$$\text{Curl Curl. } \vec{H} = 4\pi\sigma \left[\text{Curl } \vec{E} + \mu \text{ Curl} (\vec{V} \times \vec{H}) \right]$$

$$\frac{1}{4\pi\sigma\mu} \text{ Curl Curl } \vec{H} = \left[-\frac{\partial \vec{H}}{\partial t} + \text{Curl} (\vec{V} \times \vec{H}) \right]$$

$$\frac{\partial H}{\partial t} = \text{Curl}\,(V \times H) - \eta\, \text{Curl Curl}\, H$$

where $\eta = \frac{1}{4\pi\sigma\mu}$

through vector identity

$$\begin{cases} \nabla^2 H = \text{grad div}\, \vec{H} - \text{Curl Curl}\, \vec{H}. \\ \nabla^2 H = -\,\text{Curl Curl}\, \vec{H} \end{cases}$$

$$\frac{\partial H}{\partial t} = \text{Curl}\,(\vec{V} \times \vec{H}) + \eta\, \nabla^2 \vec{H}.$$

We can written another way

$$\text{Curl}\,(\vec{V} \times \vec{H}) = \vec{V}\, \text{div}\, \vec{H} - \vec{H}\, \text{div}\, \vec{V} + (\vec{H}\,\nabla)\vec{V} - (\vec{V}\,\nabla)\vec{H}$$

$$\text{Curl}\,(\vec{V} \times \vec{H}) = -H\, \text{div}\cdot \vec{V} + (\vec{H}\,\nabla)\vec{V}. - (\vec{V}\,\nabla)\vec{H}.$$

if the fluid is incompressible $\text{div}\, V = 0$

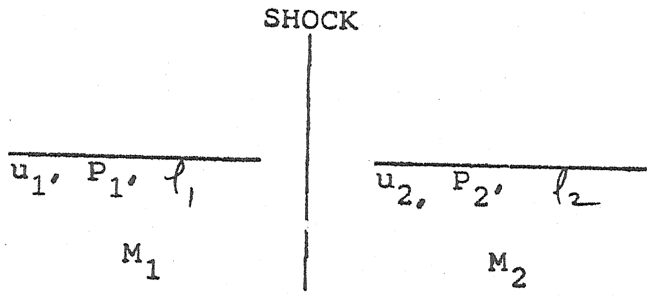
$$\text{Curl}\,(\vec{V} \times \vec{H}) = (\vec{H}\,\nabla)\vec{V} - (\vec{V}\,\nabla)\vec{H}.$$

or ,
$$\frac{\partial H}{\partial t} = (\vec{H}\,\nabla)\vec{V} - (\vec{V}\,\nabla)\vec{H} + \eta\, \nabla^2 \vec{H}$$

This gives the equation of magnetic field for incompressible viscous conducting fluid

JUMP CONDITIONS

Let a consider a normal shock to be a surface of discontinuity in velocity, pressure, density and temperature of the fluid, we shall find the relation between these quantities in front of and behind the shock. We choose the coordinate system, such that the shock is stationary and the fluid moves through it. Under these conditions in flow is steady we assume that the fluid is an ideal gas so that the perfect gas law holds both in front of and behind the shock.



To find the velocity u_2, pressure P_2, and density ρ_2 behind the shock, if the corresponding values in front of the shock are given.

Equation of continuity

$$\rho_1 u_1 = \rho_2 u_2 \tag{1.15}$$

Equation of motion

$$\rho_1 u_1 (u_1 - u_2) = p_2 - p_1 \tag{1.16}$$

Equation of energy

$$\frac{\gamma}{(\gamma-1)} \frac{p_1}{\rho_1} + \frac{u_1^2}{2} = \frac{\gamma}{(\gamma-1)} \frac{p_2}{\rho_2} + \frac{u_2^2}{2} \tag{1.17}$$

Eleminating u_2 from equation (1.15) and (1.16) we have

$$\Delta p = p_2 - p_1 = \rho_1 u_1^2 \frac{\rho_2 - \rho_1}{\rho_2}$$

$$\Delta p = \rho_1 u_1^2 \frac{\Delta \rho}{\rho_2} \qquad (1.18)$$

If we use equation (1.17) and (1.18) to eliminate u_1, we have

$$\frac{p_2 - p_1}{\rho_2 - \rho_1} = \frac{\Delta p}{\Delta \rho} = \gamma \frac{p_2 + p_1}{\rho_1 + \rho_2}$$

This is known as the Rankine- Hugoniot relation for the shock

$$\frac{dp}{d\rho} = \gamma \frac{p}{\rho}$$

We know that ratio of pressure and density

$$\frac{p_2}{p_1} = \frac{(\gamma-1)\rho_1 - (\gamma+1)\rho_2}{(\gamma-1)\rho_2 - (\gamma+1)\rho_1}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma-1)p_1 + (\gamma+1)p_2}{(\gamma-1)p_2 + (\gamma+1)p_1}$$

We have from isentropic condition

$$\frac{1}{2}u^2 + \frac{a^2}{(\gamma-1)} = \frac{a_*^2(\gamma+1)}{2(\gamma-1)}$$

then

$$\frac{1}{2}u_1^2 + \frac{a_1^2}{(\gamma-1)} = \frac{1}{2}u_2^2 + \frac{a_2^2}{(\gamma-1)} = \frac{a_*^2(\gamma+1)}{2(\gamma-1)}$$

Where a_1 and a_2 are speed of sound just behind the shock wave just after the shock wave and a_* is a critical velocity of sound ($M = \frac{q}{a}$, $q = a = a_*$)

$$\frac{p_1}{\rho_1} = \frac{a_*^2 (\gamma+1)}{2\gamma} - \left(\frac{\gamma-1}{2\gamma}\right) u_1^2 \qquad (1.19)$$

$$\frac{p_2}{\rho_2} = \frac{a_*^2 (\gamma+1)}{2\gamma} - \left(\frac{\gamma-1}{2\gamma}\right) u_2^2 \qquad (1.20)$$

Substitute the value of (1.18) and (1.19) in equation (1.17).

$$u_2 - u_1 = \frac{1}{u_1}\left[\frac{a_*^2 (\gamma+1)}{2\gamma} - \left(\frac{\gamma-1}{2\gamma}\right) u_1^2\right] - \frac{1}{u_2}\left[\frac{a_*^2 (\gamma+1)}{2\gamma} - \left(\frac{\gamma-1}{\gamma+1}\right) u_2^2\right]$$

$$u_2 - u_1 = \frac{a_*^2 (\gamma+1)}{2\gamma}\left(\frac{u_2 - u_1}{u_1 u_2}\right) + \left(\frac{\gamma-1}{2\gamma}\right)(u_2 - u_1)$$

$$\frac{a_*^2 (\gamma+1)}{2\gamma} \; \frac{1}{u_1 u_2} = \left[\frac{2\gamma - \gamma + 1}{2\gamma}\right]$$

$$\boxed{u_1 u_2 = a_*^2}$$

This relation shows that the product of velocity of gas particle just behind the shock wave just after the shock wave, u_1, u_2 will never increase to the critical velocity of sound. Let M_1 and M_2 denote the mach number just before and after the shock wave.

$$M_1 = \frac{u_1}{a_1}, \quad M_2 = \frac{u_2}{a_2}$$

$$\begin{array}{cc|cc} & P_1 & P_2 & \\ & \rho_1 & \rho_2 & \\ M_1 & T_1 & T_2 & M_2 \end{array}$$

S. W

Since by the Rankine-Hugonite relation.

$$\frac{p_2}{p_1} = \frac{(\gamma-1)\rho_1 - (\gamma+1)\rho_2}{(\gamma-1)\rho_2 - (\gamma+1)\rho_1}$$

$$\frac{p_2}{p_1} = \frac{1 - \frac{\gamma+1}{\gamma-1}\frac{\rho_2}{\rho_1}}{\frac{\rho_2}{\rho_1} - \frac{(\gamma+1)}{(\gamma-1)}}$$

from conservation law

$$\frac{\rho_1}{\rho_2} = \frac{u_2}{u_1} = m$$

Substitute it

$$\frac{p_2}{p_1} = \frac{1 - \frac{\gamma+1}{\gamma-1}\left(\frac{u_1}{u_2}\right)}{\left(\frac{u_1}{u_2}\right) - \left(\frac{\gamma+1}{\gamma-1}\right)}$$

$$\frac{p_2}{p_1} = \frac{\left\{ \frac{(r+1)}{(r-1)} \left(\frac{u_1^2}{u_1 u_2} \right) - 1 \right\}}{\left\{ \frac{(r+1)}{(r-1)} - \frac{u_1^2}{u_1 u_2} \right\}}$$

$$\frac{p_2}{p_1} = \frac{\left\{ \frac{(r+1)}{(r-1)} \frac{(r+1)}{(r-1) + \frac{2}{M_1^2}} - 1 \right\}}{\left\{ \frac{(r+1)}{(r-1)} - \frac{(r+1)}{(r-1) + \frac{2}{M_1^2}} \right\}}$$

As $u_1 u_2 = a_*^2$ As $\frac{u_1^2}{a_*^2} = \frac{r+1}{(r-1) + \frac{2}{M_1^2}}$

$$\frac{p_2}{p_1} = \frac{2 r M_1^2 - (r-1)}{(r+1)} \tag{1.22}$$

Similarly

$$\frac{\rho_2}{\rho_1} = \frac{(r-1) + (r+1)\frac{p_2}{p_1}}{(r+1) + (r-1)\frac{p_2}{p_1}}$$

$$\frac{\rho_2}{\rho_1} = \frac{\left\{ (r-1) + (r+1) \left(\frac{2 r M_1^2 - (r-1)}{(r+1)} \right) \right\}}{\left\{ (r+1) + (r-1) \left(\frac{2 r M_1^2 - (r-1)}{(r+1)} \right) \right\}}$$

$$\boxed{\frac{\rho_2}{\rho_1} = \frac{(r+1)\,M_1^{\,2}}{2+M_1^2\,(r-1)}} \qquad (1.23)$$

$$\frac{1}{2}u_1^{\,2} + \frac{a_1^{\,2}}{(r-1)} = \frac{(r+1)}{2(r-1)}a_*^{\,2}.$$

$$u_1^{\,2}\left[\frac{1}{2} + \frac{1}{(r-1)(u_1^2/a_1^2)}\right] = \frac{(r+1)}{2(r-1)}a_*^{\,2}$$

$$u_1^{\,2}\left[\frac{1}{2} + \frac{1}{(r-1)M_1^{\,2}}\right] = \frac{r+1}{2(r-1)}a_*^{\,2}$$

$$\frac{u_1^{\,2}}{a_*^{\,2}} = \frac{(r+1)}{\left\{(r-1)+\frac{2}{M_1^{\,2}}\right\}}$$

Similarly

$$\frac{u_2^{\,2}}{a_*^{\,2}} = \frac{(r+1)}{\left\{(r-1)+\frac{2}{M_2^{\,2}}\right\}}$$

divided it

$$\boxed{\frac{u_1^{\,2}}{u_2^{\,2}} = \frac{(r-1)+\frac{2}{M_2^{\,2}}}{(r-1)+\frac{2}{M_1^{\,2}}}} \qquad (1.24)$$

GENERLIZED RANKINE-HUGONIOT RELATIONS FOR MEGNETOHYDRODYNAMICS SHOCK WAVE

Shock wave under the effect of finite viscosity and thermal conductivity when the gas is conducting. When we consider a shock wave of a conducting fluid the surface of discontinuity is very thin and for a non viscous having too thermal conductivity.

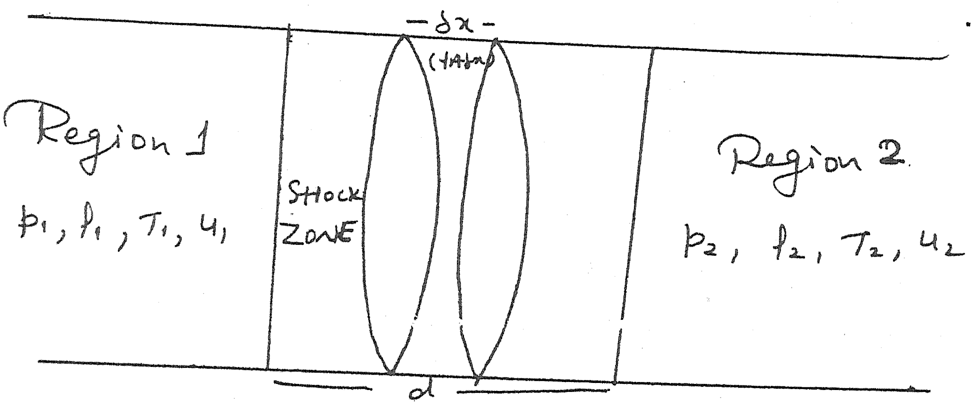
In the case of conducting shock wave under the effect of finit viscosity the surface of discontinuity has certain thickness know as shock zone. We consider a stationery shock zone of thickness in tube of uniform cross section. The flow is one dimensional and shock zone is separate in region 1 and 2.

Changes continuously from ρ_1, P_1 u_1 to $P_2, \rho_2 u_2$. This continious change from the shock zone is give to finit viscosity and thermas conductivity of gas.

Now we consider the motion of element of fixed mass moving along with the fluid particle with velocity u the flow is one dimensional.

That there is a magnetic field of intensity $H = H(x) j$ at right angle to its motion where H varies continiously from H_1 in region 1 to H_2 in region 2.

Then the pressure of magnetic field will increase the pressure by ($\mu H^2/8\pi$). The magnetic energy per unit volume of the medium is ($\mu H^2/8\pi$).



The equation of conservation of mass remains unaltered.

$$\rho_1 u_1 = \rho_2 u_2 = m.$$

The momentum equation

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

is modified to

$$p_1 + \rho_1 u_1^2 + \left(\frac{\mu H_1^2}{8\pi}\right) = p_2 + \rho_2 u_2^2 + \left(\frac{\mu H_2^2}{8\pi}\right)$$

We obtain the energy equation

$$\lambda A\,\delta x \frac{d^2T}{dx^2} = A\,\delta x \frac{d}{dx}\left(pu - \frac{4}{3}\rho\nu u\frac{du}{dx}\right) + \frac{A\rho\,\delta x}{(\gamma-1)}\,u\frac{d}{dx}\left(\frac{p}{\rho}\right) + (\rho A\,\delta x)\,u^2\frac{du}{dx}.$$

To account for the effect of the magnetic field we must modify the effective pressure from $p - \frac{4}{3}\rho\nu\,(du/dx)$ to $p + (\mu H^2/8\pi) - \frac{4}{3}\rho\nu\left(\frac{du}{dx}\right)$ and also to the right hand side we must add the rate of gain of magnetic energy of the element. Now the magnetic energy of the element of mass

$\rho A\,\delta x$ is $\quad (A\delta x)(\mu H^2/8\pi) = (A\rho\,\delta x)(\mu H^2/8\pi\rho)$

Hence the rate of increase of this magnetic energy is

$$\frac{d}{dt}\left\{(A\rho\,\delta x)\left(\frac{\mu H^2}{8\pi\rho}\right)\right\} = (A\rho\,\delta x)\frac{d}{dx}\left(\frac{\mu H^2}{8\pi\rho}\right)$$
$$= A\rho u\,\delta x\,\frac{d}{dx}\left(\frac{\mu H^2}{8\pi\rho}\right)$$

Hence the energy equation becomes

$$\lambda A\,\delta x\,\frac{d^2T}{dx^2} = A\,\delta x\,\frac{d}{dx}\left(pu + \frac{\mu H^2 u}{8\pi} - \frac{4}{3}\rho\nu\frac{du}{dx}\right) + \frac{A\rho\,\delta x}{(\gamma-1)}\,u\,\frac{d}{dx}\left(\frac{p}{\rho}\right) + (\rho A\,\delta x)u^2\frac{du}{dx}.$$
$$+ (A\rho u\,\delta x)\frac{d}{dx}\left(\frac{\mu H^2}{8\pi\rho}\right)$$

Dividing through by $\rho u A\,\delta x$ gives, since $\rho u =$ constant

$$\frac{d}{dx}\left\{\frac{\gamma}{(\gamma-1)}\frac{p}{\rho} + \frac{\mu H^2}{4\pi\rho} + \frac{1}{2}u^2 - \frac{4}{3}\nu\frac{du}{dx} - \frac{\lambda}{\rho u}\frac{dT}{dx}\right] = 0$$

integrating through the shock zone and using du/dx = 0, dT/dX = 0 in region 1, 2

$$\frac{\gamma}{(\gamma-1)}\frac{p_1}{\rho_1} + \frac{\mu H_1^2}{4\pi\rho_1} + \frac{1}{2}u_1^2 = \frac{\gamma}{(\gamma-1)}\frac{p_2}{\rho_2} + \frac{\mu H_2^2}{4\pi\rho_2} + \frac{1}{2}u_2^2$$

We thus have three equations for the four ratios p_2/p_1, u_2 / u_1, ρ_2/ρ_1, H_2 / H_1

H_2 / H_1 arises from the electremeganetic relation

$$\frac{\partial \vec{H}}{\partial t} = \nabla \times (\vec{V} \times \vec{H}) + \eta \nabla^2 \vec{H}$$

This assume that the gas in the shock zone is not a perfect conductor.

here $\partial H / \partial t = 0$ for a steady field

$\vec{H} = H(x) j$, $\vec{V} = u(x) i$

so that,

$$\vec{V} \times \vec{H} = 4 \vec{H} K$$

and

$$\nabla \times (\vec{V} \times \vec{H}) = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & u(x)H(x) \end{vmatrix} = \frac{-d(uH)}{dx} j$$

Thus $$\frac{-d(u\vec{H})}{dx} + \eta \frac{d^2 H}{dx^2} = 0$$

Integrating this through the shock zone.

$$\boxed{u_1 H_1 = u_2 H_2} \tag{1.25}$$

Hence that one of the effects of the magnetic field is to decrease the strangth ($P_2 - P_1$) / P_1 of the shock also, for a non-conducting gas the velocity in region must be greater than the speed of sound a, of the media but with the transverse field present for a donducting gas.

Shock formation required

$$u_1 > \sqrt{(a_1^2 + V_A^2)}$$

On the basis of the laws of conservation of mass, momentum and energy and maxwell's electromeganetic equations. Several types of discontinuities can exist in ideal electrically conducting fluids in the presence of maganetic fields. The discontinuities characterized by the condition that both the mass flow and density change across them are different from zero are called shock wave.

SIMILARITY CONSIDERATION

The motion of a gas or a liquid is said to be one-dimensional, the only possible one dimensional motion are produced by spherical, cylindrical, and plane wave. The simple wave solution is limited to plane wave moving in to a non uniform region. However, the problem of plane cylinedrical or spherical, Symmetrical shock wave moving in to a non uniform region are more complecated. A fairly general approximate theory for shock waves can be obtained, known as similarly solution. [7]

Then the characterstic parameter include two constants with independent dimension in addition to r and t, the partial differential equation satisfied the velocity, density and pressure in one dimensional unsteady motion of a compressible fluid. Can be replaced by ordinary differential equation for V, R and , ρ

$$v = \frac{r}{t} V \quad , \quad \rho = \frac{a}{r^{k+3} t^{5}} R ,$$

$$p = \frac{a}{r^{k+4} t^{5+2}} P$$

The solution of these ordinary differential equation can some times be obtained exactly in closed form. Such motion are called self-similar. The gas in perfect inviscid and non heat conducting, so that the motion does not involve any kind of physical or chemical change

The equation of motion, continuity and energy take the form

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} = 0 \qquad (1.27)$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial r} + (\gamma - 1) \frac{\rho v}{r} = 0 \qquad (1.28)$$

$$\frac{\partial}{\partial t}\left(\frac{p}{\rho \gamma}\right) + v \frac{\partial}{\partial r}\left(\frac{p}{\rho \gamma}\right) = 0 \qquad (1.29)$$

Where γ is the adiabatic exponent; $\gamma = 0$ for plane flow; $\gamma = 1$ for flow with cylindrical summetry; and $\gamma = 2$ for flow with spherical symmetry.

which all the flow quantities take the form $t^m f(r/t^n)$. These have simplifying feature that the partial differential equation reduce to ordinary differential equation with independent variable r/t^n. These similarty conditions originated from dimensional analysis is based on the fact that the only parameters in the problem are E with dimension ML^2/T^2 and with dimension M/L^3 the only parameter involving dimension of length and time is E/ρ_0 with dimension LS/T^2 or some function of it. Taking this is to consideration we can drive the various quantities that arise in solution [7]

THERMAL RADIATION

Radiation phenomana have acquired interest for gas mairly since attention has been attracted in Science and Technology to such phenomena as nuclear explosions, hypersonic motion of bodies in the atmosphere, powerful electric discharge and astrophysical problems, the modern trends of aerodynamics are towards high speed and high temperature as well as low density and high attitude. At higher re-entry speeds such as that for a mars probe, the interaction of aerodynamics and radiation fields becomes important radiation gasdynamics is concerved with the study of the effects of thermal radiation in a very high temperature gas flow. In radiation gas dynamics, the differential approximation for a gray gas of arbitrary opacity is largely used.

The theory of radiative transfer and radiant heat exchange was created and developed in order to understand processes which take place in stellar media and to explain the observed luminosity of stars. To as large extent this theory can be also applied to other high temperature system considered in modern physics. The thermal radiation is characterized by frequency ν of oscillation of an electromeganetic field or by the wave length λ related to the frequency and the speed of light c by the relation $\lambda = c/\nu$

If a fluid body is heated non-uniformely or if energy is released within the body, a thermal flux transported by

heat conduction appears. Heat conduction promotes energy diffusion and temperature equilization, for a weak shock, when heat conduction is present but viscosity is absent, the gas can not make a continious transfer from initial to final state. A discontinuity is necessarily formed, which corresponds to a viscous compression shock.

If the heat conduction flux is proportional to temperature gradient than all flow variable with exception of temperature undergo a discontinious jump at the discontinuity and an "isothermal shock" which are caused by electron and radiation heat conduction.

However, for extremely strong shock waves, when the energy, density and the radiation pressure become sufficiently large in comparision with the every and pressure of the fluid. The situation charges and the gas in the shock wave makes a continious transition from the initial to final state through radiation conduction alone [8] [9] [10]

(i) RADIATION PRESSURE

Which may be expressed as

$$P_R = \frac{1}{3} \quad a \quad T^4 \qquad (1.30)$$

Where T is the temperature of the gas and 'a' is known as the Stefan-Boltzmann constant. Total pressure at each point of flow field is equal to the sum of gas pressure and radiation pressure.

(ii) RADIATION ENERGY DENSITY

The radiation energy density per unit mass of the fluid is given by

$$\frac{E_R}{\rho} = \frac{aT^4}{\rho} \tag{1.31}$$

Where ρ is the density of the fluid.

(iii) RADIATION FLUX F

The radiation flux F is given by the formula,

$$F = \frac{ac}{4} \frac{T^4}{\rho} \tag{1.32}$$

Where c is the velocity of light in vaccum. By taking into account the effects of radiation and considering the gas to be inviscid and non heat conducting.

The equation of motion and energy

$$\rho \frac{D\vec{q}}{Dt} = -\operatorname{grad}(p + p_R) \tag{1.33}$$

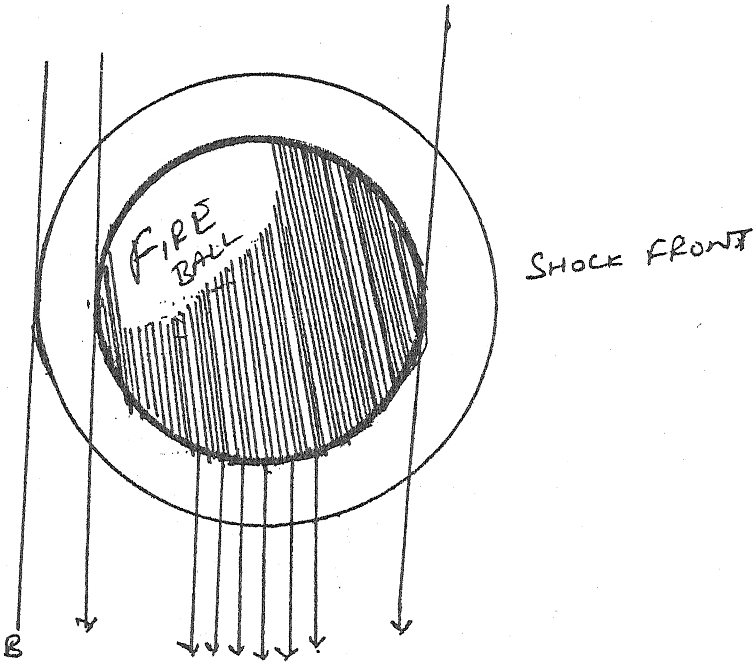
and

$$\rho \frac{De_m}{Dt} = \operatorname{div}(pq) + \operatorname{div} F \tag{1.34}$$

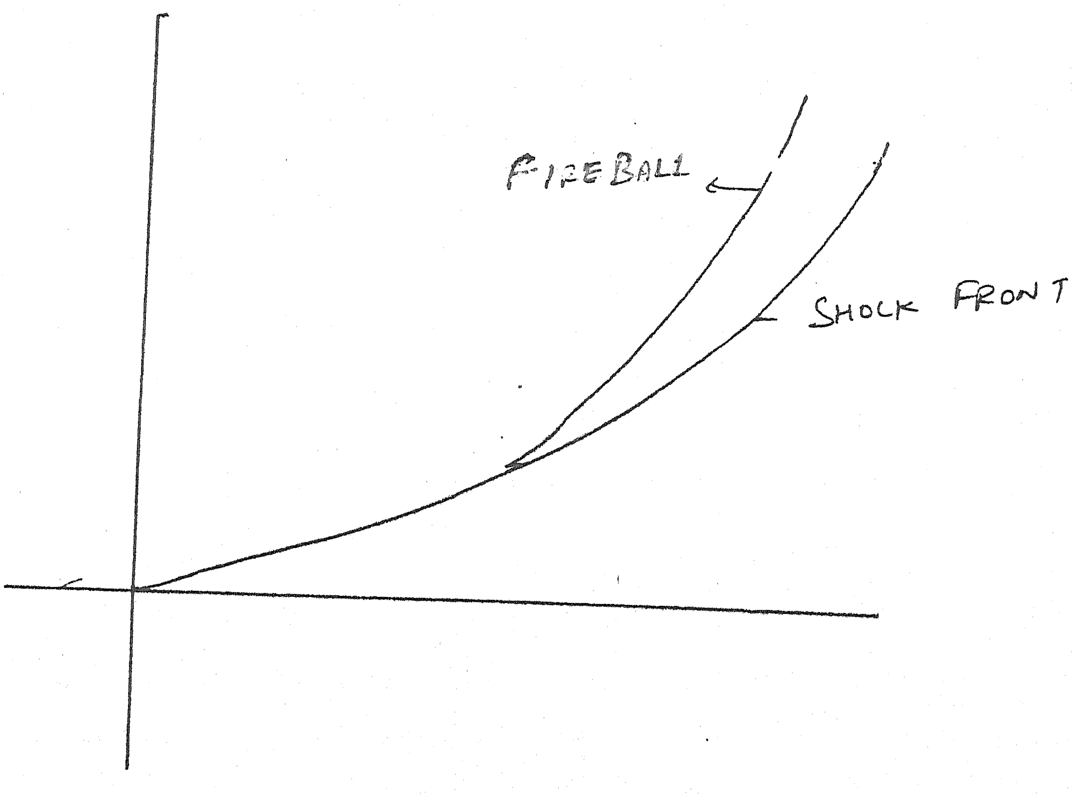
where

$$e_m = e + \frac{1}{2} q^2 + \frac{E_R}{\rho} \tag{1.35}$$

and is the total energy per unit mass



Schematic representation of the luminescence of a fire ball after brekamay. The inner circle is boundary of the luminous mass, the fire ball, the outer circle is the shock front.



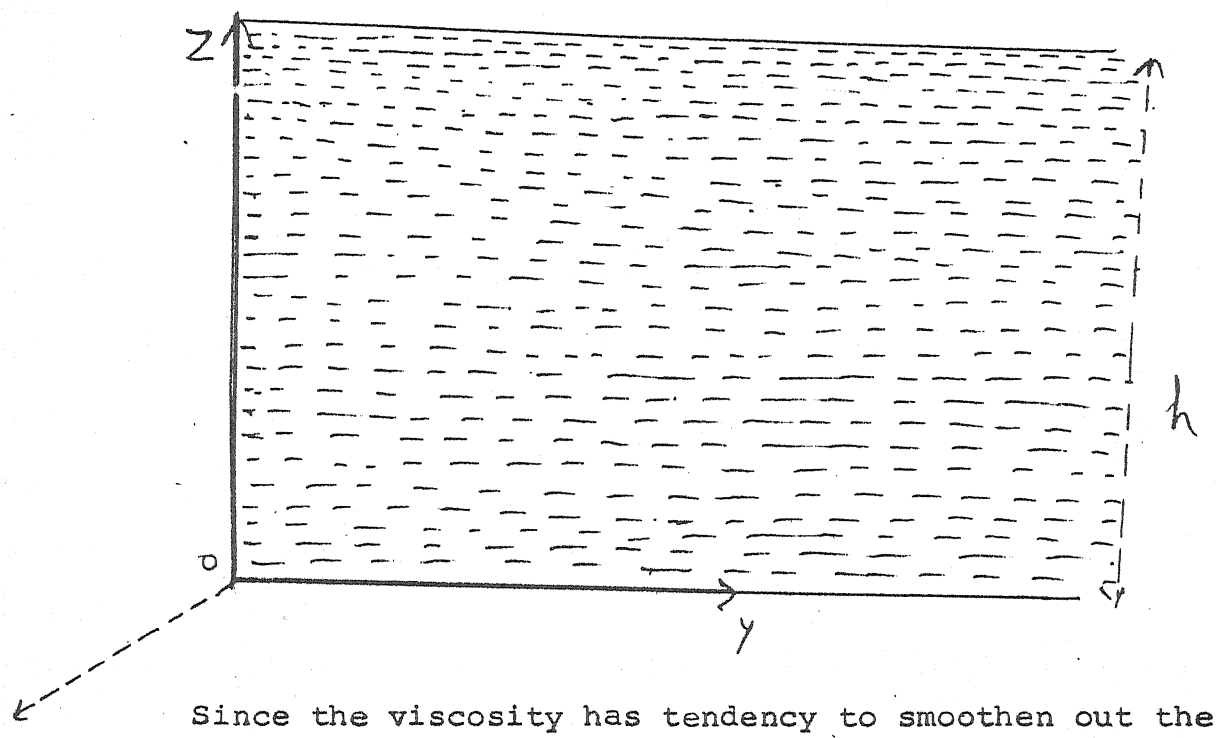
VISCOSITY

Consider in the real fluid, the surface of discontinuity should be replaced by transition region. If the thickness of the transition region is very thin so that we are interested in the flow inside the transition region or in flow phenomena closely related to the transition region; we must consider the effect of viscosity.

In many problem of fluid flow, the fluid is actually a mixture of several gases, i.e. air is a mixture of oxygen, nitrogen, and other gases. If the concentration of the gases in the mixture remain unchanged during the flow field, we may consider the mixture as a single fluid.

Viscosity represents that property of an actual fluid which exhibits a certain resistance to alteration of form. Although this resistance is comparatively small for many practically important fluids, such as water or gases, it is not negligible, for other fluids such as oil, glycerine etc., this resistance is quit large. In a viscous fluid, both tangential and normal forces exist. Some of the kinetic energy of flow will be dissipated as heat through the viscous forces.

First introduced the concept of the boundary layer by stating that the viscous effects are confined to a very thin layer near the boundary. The viscous effects are confined to very thin boundary layer whose thickness tends to be zero as the coefficient of kinematic viscosity goes to zero. The Navier-stoks equation can be simplified by such consideration.



Since the viscosity has tendency to smoothen out the discontinuity due viscous forces applied against the layers of fluid, therefore, the equation of motion, as such, can not be taken for viscous fluid.

However, Richtmyer and Von-Nenumann [11] suggested the artificial viscosity, in terms of velocity gradient applicable for strong shocks only which follows the dimensional analysis so that equation of motion can be transformed to numerically integrable differentiable equation. The term suggested by Richtmyer and Von-Nenumann is given as

$$q = \frac{1}{2} k^2 + r^2 \frac{\partial u}{\partial r}\left(\left| \frac{\partial u}{\partial r} \right| - \frac{\partial u}{\partial r} \right)$$

SPHERICAL SHOCK WAVES IN VISCOUS MAGNETO GAS DYNAMICS

****

The resistance of shock waves gasdynamics flow field introduce free boundary discontinuities in to Physical Parameter of the system. A method for avoding such difficulties, particularly for the numerical calculations, has been developed by Richtmyer and Von Neumann.

They observed that the addition of a particular viscosity like term into gas dynamics equation could lead to continious shock flow field in which the finite thickness of discontinuities at theshock wave was removed and replaced by a region in which the Physical Parameter change rapidly, but smoothly.

In the present work, the artificial mechanism of viscosity in the presence of magnetic field to smear out discontinuities of the physical parameters from the flow field. In order to give a meaning to the other wise physically unrealisable magnetic field with the spherical symmetry, the magnetic field is replaced by an idealized field such that lines of forces lie on a hemisphere whose centre is the point of explosion. We have used the Runga-Kutta method to obtain numerical solutions in viscous and inviscid regions. We have shown that field variable change rapidly when the magnetic field is imposed in both the viscous

and the non-viscous regions.

The equations of motion of a fluid having infinite electrical conductivity with artificial viscosity and expressed in spherically symmetric form, are,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial}{\partial r}(p+q) + \frac{H}{\rho}\frac{\partial H}{\partial r} + \frac{H^2}{\rho r} = 0 \qquad (2.1)$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right) = 0 \qquad (2.2)$$

$$\frac{\partial H}{\partial t} + u\frac{\partial H}{\partial r} + H\frac{\partial u}{\partial r} + \frac{Hu}{r} = 0 \qquad (2.3)$$

and,

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r} - \frac{\{\gamma p + (\gamma-1)q\}}{\rho}\left\{\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right\} = 0 \qquad (2.4)$$

Where u is fluid velocity, ρ be density, H be component of transverse magnetic field and q is artificial viscosity.

We seek the solution of equation in the form

$$p(r,t) = \rho_0 R^{-\beta} f(x),$$

$$\rho(r,t) = \rho_0 \psi(x),$$

$$u(r,t) = R^{-\alpha} \phi(x)$$

and the non-viscous regions.

The equations of motion of a fluid having infinite electrical conductivity with artificial viscosity and expressed in spherically symmetric form, are,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial}{\partial r}(p+q) + \frac{H}{\rho}\frac{\partial H}{\partial r} + \frac{H^2}{\rho r} = 0 \qquad (2.1)$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right) = 0 \qquad (2.2)$$

$$\frac{\partial H}{\partial t} + u\frac{\partial H}{\partial r} + H\frac{\partial u}{\partial r} + \frac{Hu}{r} = 0 \qquad (2.3)$$

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Where u is fluid velocity, ρ be density, H be component of transverse magnetic field and q is artificial viscosity.

We seek the solution of equation in the form

$$p(r,t) = p_0 R^{-\beta} f(x),$$

$$\rho(r,t) = \rho_0 \psi(x),$$

$$u(r,t) = R^{-\alpha} \phi(x)$$

and

$$H(r,t) = \sqrt{t_0 R^{-\beta}}\, \eta(x)$$

Where $x = \frac{r}{R}$, R being a function of time only we assumed that,

$$\alpha = \tfrac{1}{2}\beta \quad \text{and} \quad R^{\alpha} R' = A \text{ (constant)}$$

$$u(r,t) = R^{-\alpha} \phi(x)$$

$$\frac{\partial u}{\partial t} = -\alpha R^{-\alpha-1} \frac{dR}{dt} \phi(x) + R^{-\alpha} \phi'(x) \frac{\partial x}{\partial t}$$

$$\frac{\partial u}{\partial t} = -\alpha R^{-\alpha-1} R' \phi(x) + R^{-\alpha} \phi'(x) \left(\frac{-x R'}{R}\right)$$

$$\frac{\partial u}{\partial t} = -R^{-\alpha-1} R' \left[\alpha\phi + x\phi'(x)\right]$$

$$\frac{\partial u}{\partial t} = -R^{-\alpha-1} A R^{-\alpha} \left[\alpha\phi + x\phi'\right]$$

As $\{R' R^{\alpha} = A\}$

$$\frac{\partial u}{\partial t} = -R^{-2\alpha-1} A \left[\alpha\phi + x\phi'\right] \quad \text{— [2.5]}$$

$$\frac{\partial u}{\partial r} = R^{-\alpha} \phi'(x) \frac{\partial x}{\partial r}$$ As R is the function of x only

$$\frac{\partial u}{\partial r} = R^{-\alpha} \phi'(x) \frac{1}{R}$$

$$\frac{\partial u}{\partial r} = R^{-\alpha-1} \phi'(x) \quad \text{— [2.6]}$$

$$p + q = t_0 R^{-\beta} f(x) + \tfrac{1}{2} k^2 \rho r^2 \frac{\partial u}{\partial r}\left(\left|\frac{\partial u}{\partial r}\right| - \frac{\partial u}{\partial r}\right)$$

$$p+q = \rho_0 R^{-\beta} f(x) + \tfrac{1}{2} K^2 \rho_0 \psi(x) r^2 R^{-\alpha-1} \Phi'(x)$$
$$\times R^{-\alpha-1} (|\Phi'| - \Phi')$$

or, put the value of (r = u R)

$$p+q = \rho_0 R^{-\beta} f(x) + \tfrac{1}{2} K^2 \rho_0 \psi(x) x^2 R^2 R^{-2\alpha-2}$$
$$\times \Phi' (|\Phi'| - \Phi')$$

$$p+q = \rho_0 R^{-\beta} f(x) + \frac{K^2}{2 R^{2\alpha}} \psi(x) \rho_0 x^2 \Phi'(|\Phi'|-\Phi')$$

$$p+q = \rho_0 R^{-\beta} f(x) + g(x) \frac{\rho_0}{R^{2\alpha}}$$

$$\text{As} \left\{ g(x) = \frac{K^2}{2} \psi(x) x^2 \Phi' (|\Phi'| - \Phi') \right\}$$

Differentiate it,

$$\therefore \frac{\partial}{\partial r}(p+q) = \rho_0 R^{-\beta} f'(x) \frac{1}{R} + g'(x) \frac{1}{R} \frac{\rho_0}{R^{2\alpha}}$$

$$\frac{\partial}{\partial r}(p+q) = \rho_0 R^{-\beta-1} f'(x) + \rho_0 R^{-2\alpha-1} g'(x)$$

$$\frac{\partial}{\partial r}(p+q) = \rho_0 R^{-2\alpha-1} f'(x) + \rho_0 R^{-2\alpha-1} g'(x)$$

$$\text{As} \left\{ \alpha = \tfrac{1}{2}\beta \right\}$$

$$\frac{\partial}{\partial r}(p+q) = \rho_0 R^{-2\alpha-1} (f' + g') \quad \text{--- [2-7]}$$

We have,

$$H = \sqrt{\rho_0 R^{-\beta}}\ \eta(x)$$

$$\frac{\partial H}{\partial r} = \sqrt{\rho_0 R^{-\beta}}\ \eta'(x)\ \frac{\partial x}{\partial r}$$

$$\frac{\partial H}{\partial r} = \sqrt{\rho_0}\ R^{-\beta/2}\ \frac{1}{R}\ \eta'$$

$$\text{As } \{ x = r/R \}$$

$$\frac{\partial H}{\partial r} = \rho_0^{1/2}\ R^{-\beta/2 - 1}\ \eta'$$

$$\frac{\partial H}{\partial r} = \rho_0^{1/2}\ R^{-\alpha-1}\ \eta' \quad — [2.8]$$

Substitute the value from the equation (2.5), (2.6), (2.7), (2.8) in equation (2.1) we get

$$- R^{-2\alpha-1} A(\alpha\phi + x\phi') + R^{-\alpha}\phi + R^{-\alpha-1}\phi'$$

$$+ \frac{\rho_0 R^{-2\alpha-1}}{\rho_0\ \psi(x)}(f' + g') + \frac{\rho_0 R^{-\beta}\eta(x)\ \eta(x)}{\rho_0\ x\ R\ \psi(x)}$$

$$+ \frac{\rho_0^{1/2}\ R^{-\beta/2}\ \eta(x)}{\rho_0\ \psi(x)}\ \rho_0^{1/2}\ R^{-\alpha-1}\eta'$$

$$= 0$$

or,

$$- R^{-2\alpha-1} A(\alpha\phi + x\phi') + R^{-2\alpha-1}\phi\phi' + R^{-2\alpha-1}\frac{f'+g'}{4} + R^{-2\alpha-1}\frac{\eta\eta'}{4} + R^{-2\alpha-1}\frac{\eta^2}{4x} = 0$$

$$\boxed{-A(\alpha\phi + x\phi') + \phi\phi' + \frac{f'+g'+\eta\eta'}{4} + \frac{\eta^2}{4x} = 0} \quad \text{—(A)}$$

where

$$\frac{g(x)}{R^{2\alpha}} = \frac{k^2}{2R^{2\alpha}} 4 x^2 \phi'(|\phi'| - \phi')$$

$$f = f_0 \psi(x)$$

Differentiating with respect to t

$$\frac{\partial f}{\partial t} = f_0 \psi'(x) \frac{\partial x}{\partial t}$$

$$\frac{\partial f}{\partial t} = f_0 \psi'(x) \frac{xR'}{R}$$

$$\frac{\partial f}{\partial t} = -A R^{-\alpha-1} x f_0 \psi'(x)$$

Differentiation with respect to r

$$\frac{\partial \rho}{\partial r} = \rho_0 \ \psi'(x) \ \frac{1}{R}$$

Substitute the value of $\frac{\partial \rho}{\partial t}$, $\frac{\partial \rho}{\partial r}$, $\frac{\partial u}{\partial r}$, ρ

and u in (2.2) we get

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \left(\frac{\partial u}{\partial r} + \frac{2u}{r} \right) = 0$$

$$-A R^{-\alpha-1} x \rho_0 \psi'(x) + R^{-\alpha} \phi(x) \rho_0 \psi'(x) \frac{1}{R}$$
$$+ \rho_0 \psi(x) \left(R^{-\alpha-1} \phi'(x) + \frac{2 R^{-\alpha-1} \phi(x)}{x} \right) = 0$$

Or, $$-A R^{-\alpha-1} x \rho_0 \psi' + R^{-\alpha-1} \rho_0 \phi \psi'$$
$$+ R^{-\alpha-1} \rho_0 \psi \left(\phi' + \frac{2\phi}{x} \right) = 0$$

Or, $$-A R^{-\alpha-1} x \rho_0 \psi' + R^{-\alpha-1} \rho_0 \phi \psi'$$
$$+ R^{-\alpha-1} \rho_0 \psi \left(\phi' + \frac{2\phi}{x} \right) = 0$$

Or, $$-xA \psi' + \phi \psi' + \psi \left(\phi' + \frac{2\phi}{x} \right) = 0$$

Or, $$\boxed{\psi'(\phi - Ax) + \psi \phi' + \frac{2\phi\psi}{x} = 0} \quad \text{— (B)}$$

we have

$$H = \sqrt{\rho_0 R^{-\beta}} \, \eta(x) = \rho_0^{1/2} R^{-\beta/2} \eta(x)$$

$$\frac{\partial H}{\partial t} = -\rho_0^{1/2}\, \beta/2\; R^{-\beta/2-1}\, y(x)\, R' + \rho_0^{1/2}\, R^{-\beta/2}\, y'(x)\, \frac{\partial x}{\partial t}$$

$$\frac{\partial H}{\partial t} = -\frac{\beta}{2}\, \rho_0^{1/2}\, R^{-\beta/2-1}\, y(x)\, R' - \rho_0^{1/2}\, R^{-\beta/2-1}\, y' x R'$$

$$\frac{\partial H}{\partial t} = -\rho_0^{1/2}\, R^{-\beta/2-1}\, R'\, (\beta/2\, y + y' x)$$

$$\frac{\partial H}{\partial t} = -\rho_0^{1/2}\, R^{-2\alpha-1}\, A\, (\alpha y + x y')$$

Also,

$$\frac{\partial H}{\partial r} = \rho_0^{1/2}\, R^{-\alpha-1}\, y'$$

and we have

$$\frac{\partial u}{\partial r} = R^{-\alpha-1}\, \phi'(x)$$

Put these values in equation (2.5), we get

$$-\rho_0^{1/2}\, R^{-2\alpha-1}\, A\, (\alpha y + x y') + R^{-\alpha}\phi(x)\rho_0 R^{-\alpha} y' + \rho_0^{1/2}\, R^{-2\alpha-1}\, y(x)\, \phi'(x) + \frac{R^{-\alpha}\, \phi(x)\, \rho_0^{1/2}\, R^{-\alpha}\, y(x)}{x R} = 0$$

or, $-A(\alpha y + x y') + \phi y' + y \phi' + \frac{\phi y}{x} = 0$

$$\boxed{y'(\phi - Ax) + y\left(\phi' - A\alpha + \frac{\phi}{(x)}\right) = 0} \quad \text{— (c)}$$

we have

$$p(r,t) = p_0 R^{-\beta} f(x)$$

$$\frac{\partial p}{\partial t} = p_0(-\beta) R^{-\beta-1} R' f(x) + p_0 R^{-\beta} f'(x) \frac{\partial x}{\partial t}$$

$$\frac{\partial p}{\partial t} = -p_0 \beta R^{-\beta-1} A R^{-\alpha} f(x) - p_0 \frac{x A R^{-\alpha}}{R} R^{-\beta} f'(x)$$

$$\frac{\partial p}{\partial t} = -p_0 R^{-\beta-\alpha-1} A (\beta f(x) + x f'(x))$$

$$\frac{\partial p}{\partial t} = -p_0 R^{-3\alpha-1} A (2\alpha f + x f')$$

As $[\alpha = \beta/2]$

Also

$$\frac{\partial p}{\partial r} = p_0 \, R^{-\beta} f'(x) \frac{\partial x}{\partial r}$$

$$\frac{\partial p}{\partial r} = p_0 \, R^{-\beta-1} f'(x)$$

$$\frac{\partial p}{\partial r} = p_0 \, R^{-2\alpha-1} f'$$

$$\rho = \rho_0 \, \psi(x)$$

$$\frac{\partial \rho}{\partial t} = \rho_0 \, \psi'(x) \frac{\partial x}{\partial t}$$

$$\frac{\partial \rho}{\partial t} = -\frac{x R'}{R} \rho_0 \, \psi'(x)$$

$$\frac{\partial \rho}{\partial t} = -A \, R^{-\alpha-1} \cdot x \, \rho_0 \, \psi'(x)$$

$$\frac{\partial \rho}{\partial r} = \rho_0 \, \psi'(x) \, R^{-\alpha}.$$

Put these value in equation (2.6)

$$- \rho_0 \, R^{-3\alpha-1} A \left(2\alpha f + x f' \right) + R^{-\alpha} f(x) \rho_0 \, R^{-2\alpha-1} f'$$

$$- \frac{\left\{ \gamma \, p_0 \, R^{-2\alpha} f(x) + (\gamma - 1) \, \rho_0 \, R^{-2\alpha} g \right\}}{\rho_0 \, \psi(x)} \times$$

$$\times \left(-A \, R^{-\alpha-1} x \, \rho_0 \, \psi' + \rho_0 \, R^{-\alpha-1} \psi' f \right)$$

$$= 0$$

or, $-A\,(2\alpha f + x f') + \phi f' - \dfrac{\gamma f + (\gamma-1) g}{\psi}\left(-A x \psi' + \phi \psi'\right) = 0$

or,

$$\boxed{-2A\alpha f + f'(\phi - Ax) - \frac{\gamma f + (\gamma-1) g}{\psi} \times \psi'(\phi - Ax) = 0} \quad \text{—(D)}$$

If E is the total energy then the total energy is given by

$$E = 4\pi \int_0^\infty r^2 \left(\rho \frac{u^2}{2} + \frac{P}{(\gamma-1)} + \frac{H^2}{2} \right) dr.$$

Put the value ρ, u, P, H in equation we get,

$$E = 4\pi \int_0^\infty r^2 \left(\frac{1}{2} \rho_0 \psi(x) R^{-2\alpha} \phi^2 + \frac{\rho_0 R^{-\beta} f(x)}{(\gamma-1)} + \frac{\rho_0 R^{-\beta} h^2}{2} \right)$$

$$E = 4\pi \int_0^\infty x^2 R^2 \left(\frac{1}{2} \rho_0 R^{-2\alpha} \psi \phi^2 + \frac{\rho_0 R^{-2\alpha} f(x)}{(\gamma-1)} + \frac{1}{2} \rho_0 R^{-2\alpha} h^2 \right) R\, dx$$

$$E = 4\pi \int_0^\infty \rho_0 \, R^{-2\alpha+3} \left(\frac{1}{2} \psi \phi^2 + \frac{f}{(\gamma-1)} + \frac{1}{2} \eta^2 \right) x^2 dx.$$

$$\boxed{E = 4\pi \rho_0 R^{3-2\alpha} \int_0^\infty \left(\frac{1}{2} \psi \phi^2 + \frac{f}{(\gamma-1)} + \frac{1}{2} \eta^2 \right) x^2 dx} \quad \text{---}(E)$$

We now introduce new dimension less quantities

$$F = \frac{f}{A}, \quad G = \frac{g}{A}, \quad P = \frac{\phi}{A}, \quad N = \frac{\eta}{A}.$$

and also $\alpha = 3/2$

The equation (A) – (E) transformed in to the form

$$-\left(\frac{3}{2} \frac{\phi}{A} + x \frac{\phi'}{A} \right) + \frac{\phi}{A} \frac{\phi'}{A} + \frac{\frac{f'}{A2} + \frac{g'}{A2} + \frac{\eta}{A}\frac{\eta'}{A}}{4}$$

$$+ \frac{\eta^2/A}{4x} = 0$$

or,

$$\boxed{-\left(\frac{3}{2} P + x P' \right) + P P' + \frac{F' + G' + N N'}{4} + \frac{N^2}{4x} = 0}$$

Divided whole equation (B.) by A we get

$$4 \left(\frac{\phi}{A} - x \right) + 4 \frac{\phi'}{A} + \frac{2 \, 4 \, \phi/A}{x}.$$

$$\boxed{\psi'(P-x) + \psi P' + \frac{2\psi P}{x} = 0} \quad -(I)$$

Divided whole equation (C) by A^2 and put $\alpha = 3/2$, we get

$$\frac{\eta'}{A}\left(\frac{f}{A} - x\right) + \frac{\eta}{A}\left(\frac{f'}{A} - \frac{3}{2} + \frac{f/A}{x}\right) = 0$$

$$\boxed{N'(P-x) + N\left(P' - \frac{3}{2} + \frac{P}{x}\right) = 0} \quad -(J)$$

Divided whole equation (D) by A^2

$$-2 \times \frac{3}{2}\frac{f}{A} + \left(\frac{f}{A} - x\right)\frac{f'}{A} - \frac{\gamma f/A + (\gamma-1)g/A}{\psi} \times$$

$$\times (f/A - x)\psi' = 0$$

or,

$$\boxed{-3F + (P-x)F' - \frac{\gamma F + (\gamma-1)G}{\psi}(P-x)\psi' = 0} \quad -(K)$$

We have equation,

$$\frac{g}{A^2} = \frac{1}{2} k^2 \psi x^2 \frac{\Phi'}{A}\left(\left|\frac{\Phi'}{A}\right| - \frac{\Phi'}{A}\right)$$

$$\boxed{G = \frac{1}{2} k^2 \psi x^2 p' (|p'| - p')} \quad \text{--- (Q)}$$

and equation (E) can be written as,

$$E = 4\pi \rho_0 R^{3-2\alpha} A^2 \int_0^\infty \left(\frac{\psi \Phi^2/A^2}{2} + \frac{f/A^2}{(r-1)} + \frac{\eta^2/A^2}{2} \right) x^2 dx.$$

$$\boxed{E = 4\pi \rho_0 R^{3-2\alpha} A^2 \int_0^\infty \left(\frac{\psi p^2}{2} + \frac{f}{(r-1)} + \frac{N^2}{2} \right) x^2 dx} \quad \text{--- (W)}$$

In the viscous region $x \geq 1$

When $p' \leq 0$ the equation (Q) may be written as

$$\boxed{p' = -\frac{1}{kx}\left(\frac{G}{4}\right)^{1/2}} \quad \text{--- (L)}$$

(H), (I), (J), (K) and (L) represent the equation in the viscous region $x \geq 1$ for the region without viscosity $(0 \leq x \leq 1)$ q = 0 i.e. G = 0 the equation reduces to,

$$-\left(\frac{3}{2} P + x P'\right) + P P' + \frac{F' + N N'}{\psi} + \frac{N^2}{4x} = 0 \qquad \text{(M)}$$

$$\psi'(P - x) + \psi P' + 2\psi \frac{P}{x} = 0 \qquad \text{(N)}$$

$$N'(P - x) + N\left(P' + \frac{P}{x} - \frac{3}{2}\right) = 0 \qquad \text{(O)}$$

and,

$$-3F + (P - x)F' - \gamma F (P - x)\frac{\psi'}{\psi} = 0 \qquad \text{(R)}$$

Since,

$$H = \sqrt{\rho_0 R^{-\beta} \eta}$$

$$\frac{H}{A} = \sqrt{\rho_0}\; R^{-\beta/2} \eta)/A = \sqrt{\rho_0}\; R^{-\alpha} N$$

or,

$$N = \frac{H}{A R^{-\alpha} \sqrt{\rho_0}}$$

$$N = \frac{H}{R' \sqrt{\rho_0}} \qquad \text{As} \quad R^{\alpha} R' = A \ (\text{constant})$$

$$N = \frac{H}{V \sqrt{\rho_0}} \qquad \text{As} \quad V = \frac{dR}{dt} = R'$$

$$N = \frac{1}{M_A}$$

The equation are solved numerically for the viscous region $(x \geq 1)$

The jump conditions are given by

$$\frac{\rho_1}{\rho_0} = \frac{\gamma+1}{\gamma-1}, \quad \frac{p_1}{p_0} = \frac{2}{\gamma+1}, \quad \frac{u_1}{u_0} = \frac{2}{\gamma+1}$$

$$\frac{H_1}{H_0} = \left(\frac{\gamma+1}{\gamma-1}\right) \frac{1}{M_A}.$$

Using the similarity variable

$$P(1) = \frac{2}{\gamma+1}, \quad F(1) = \frac{2}{(\gamma+1)},$$

$$\psi(1) = \frac{(\gamma+1)}{(\gamma-1)}, \quad N(1) = \frac{(\gamma+1)}{(\gamma-1)} \frac{1}{M_A}$$

Let $\delta = \frac{\rho_0}{\rho}$ is the ratio of densities just ahead and just behind the shock front

So
$$P(1) = (1-\delta), \quad F(1) = (1-\delta),$$

$$\psi(1) = \frac{1}{\delta}, \quad N(1) = \frac{M_A^{-1}}{\delta}$$

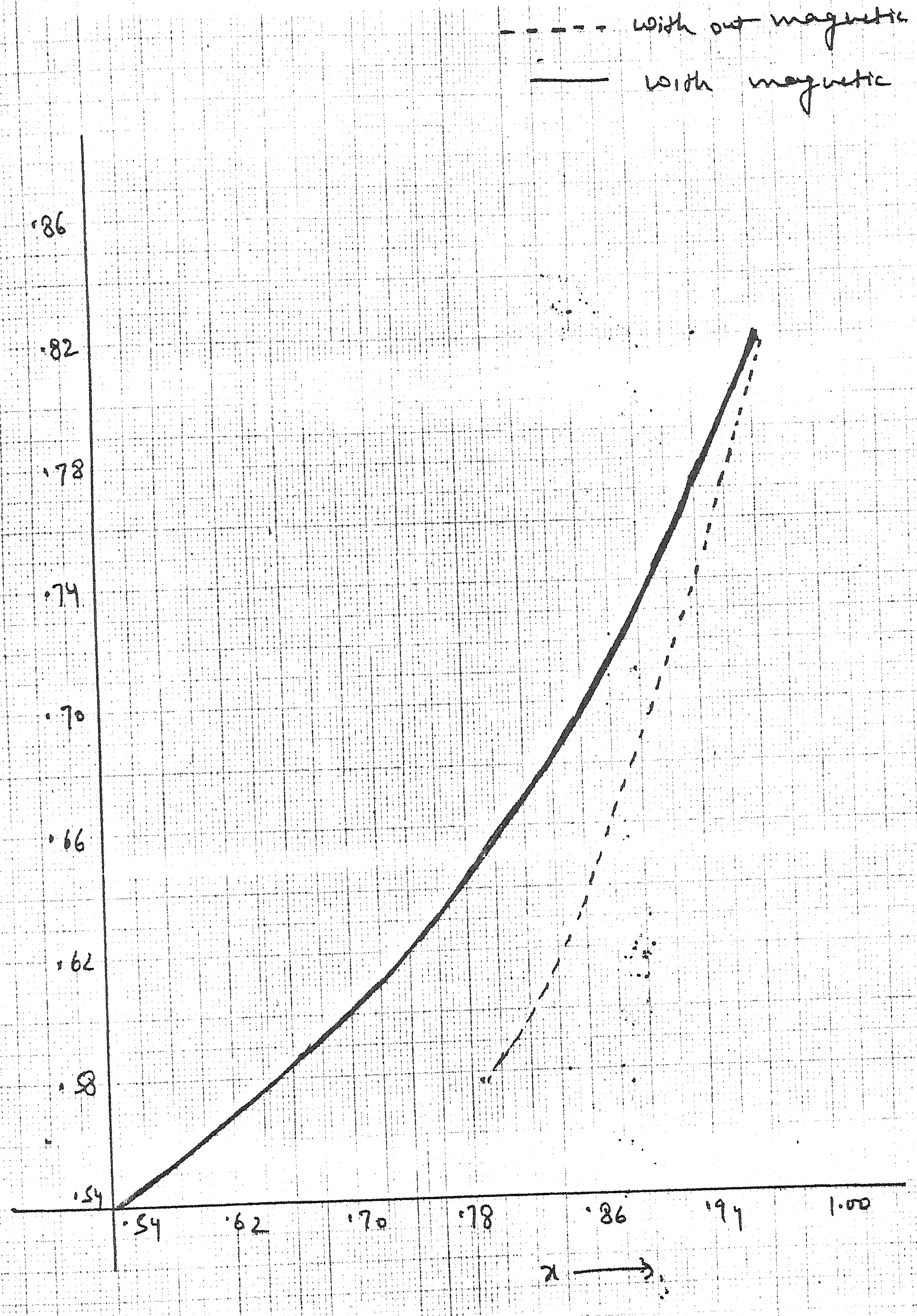
CONCLUSION

The equation (H), (I), (J), (K) and (L) are solved numerically using boundary conditions for the viscous region ($x \geq 1$) for which $\rho' \leq 0$ and the numerical solution of equation (M), (N), (O), and (R) are obtained in the region without viscosity ($0 \leq x \leq 1$)

We conclude that the field parameters change rapidly in the inviscid region as well as in viscous region when the magnetic field is imposed. The variation of velocity, pressure, density, magnetic field and viscosity for both the viscious and non-viscous region has been illustrated through the graph 1 to 9. We also infer that in viscous region, the field parameters attain a maximum value at the shock front when the artificial mechanism of viscosity is weaker; i.e. for smaller value of $K = 0.0349$ and starts decreasing rapidly was we move away from the shock front except the pressure which decreases rapidly only for a narrow region and the increases instantaneously. The magnetic field has the significant effect on the flow parameters for this value of K but for $K = 0.349$.

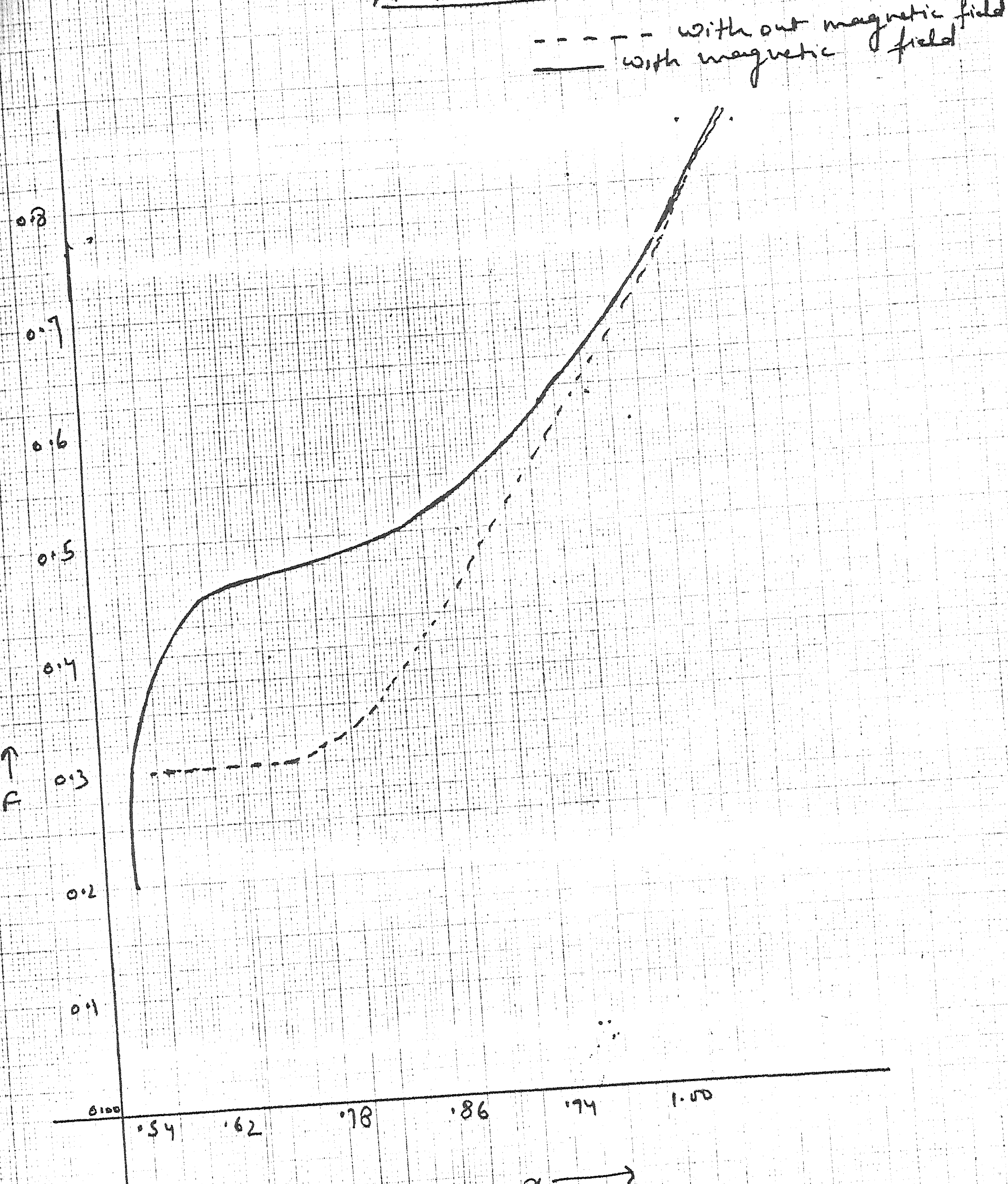
The effect of magnetic field does not illustrate any significant change in flow parameters. This shows that the effect of magnetic field does not play any important role when the artificial mechanism of viscosity is stronger.

Non viscous region:



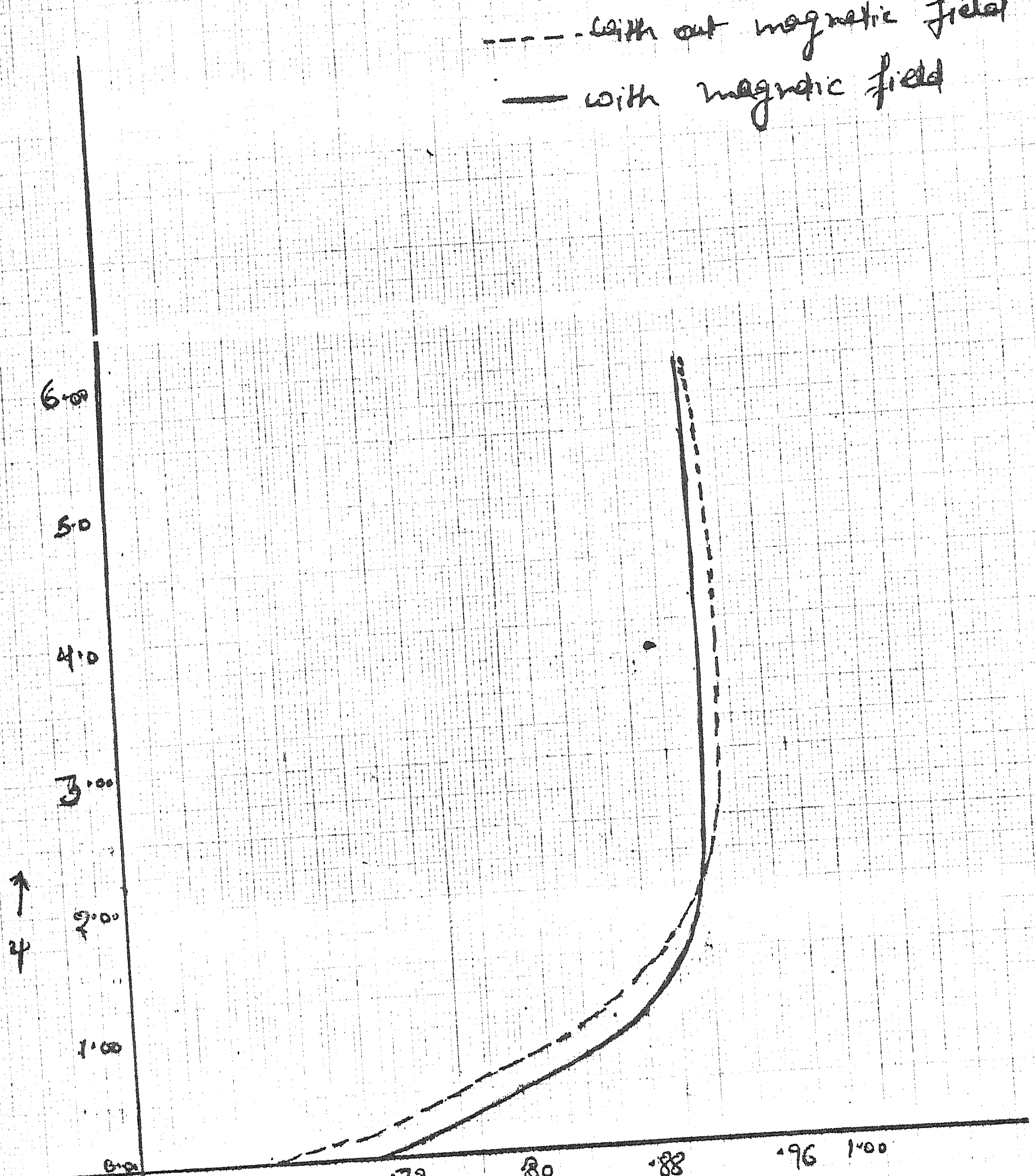
Velocity distribution:

Non viscous Region.



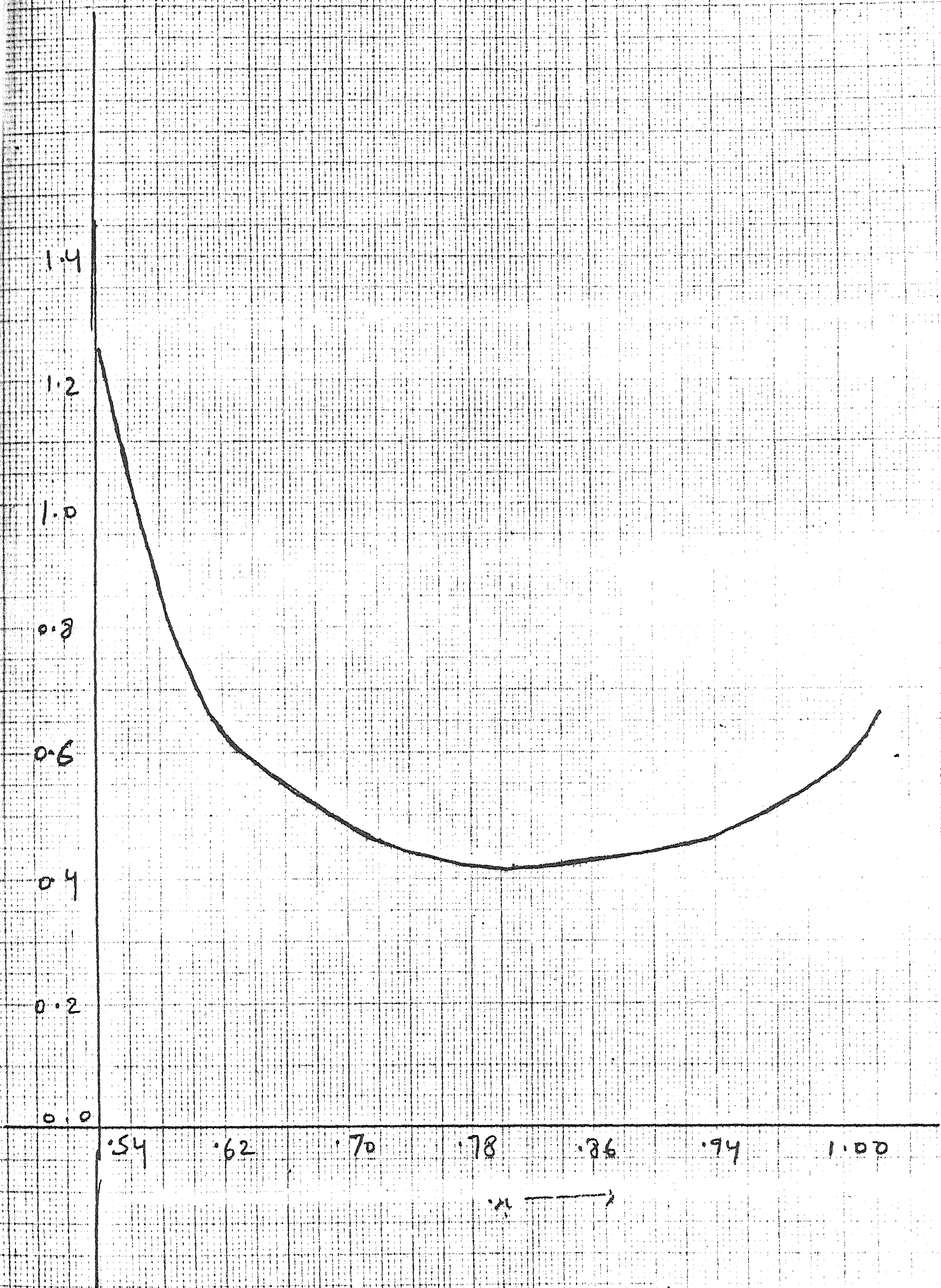
Pressure distribution.

Non Viscous Region



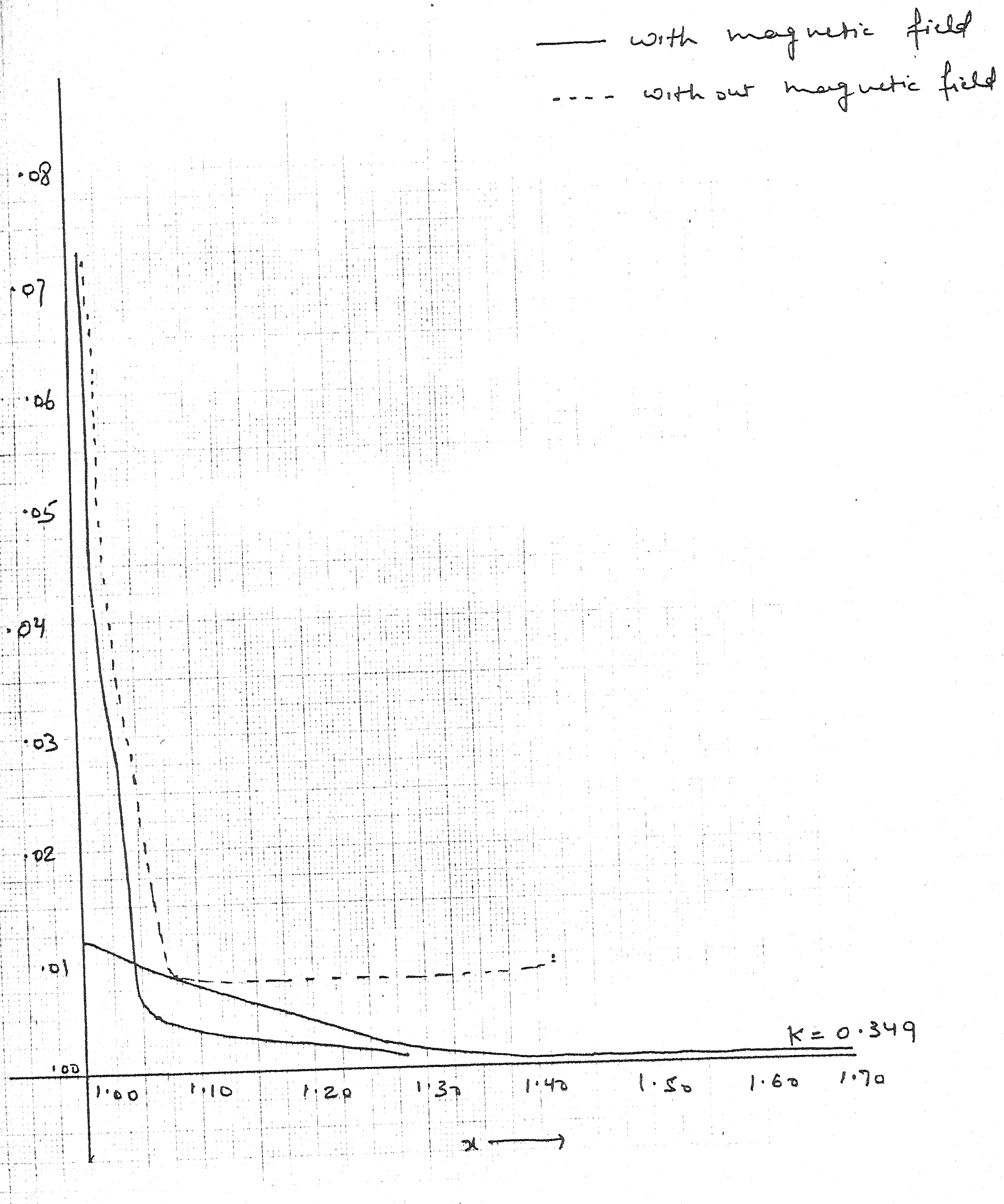
Density distribution

Non-viscous region



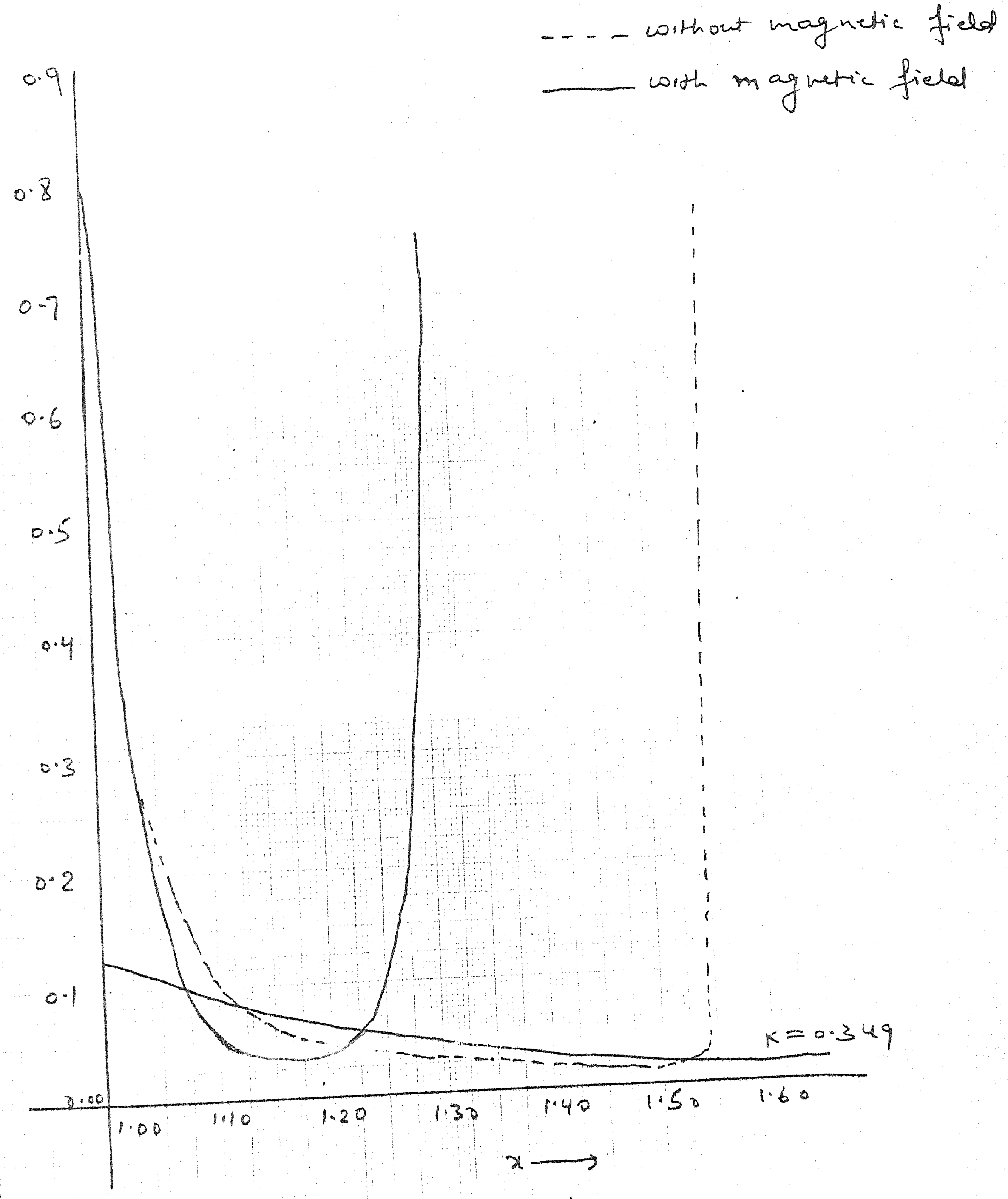
Magnetic field distribution

Viscous Region



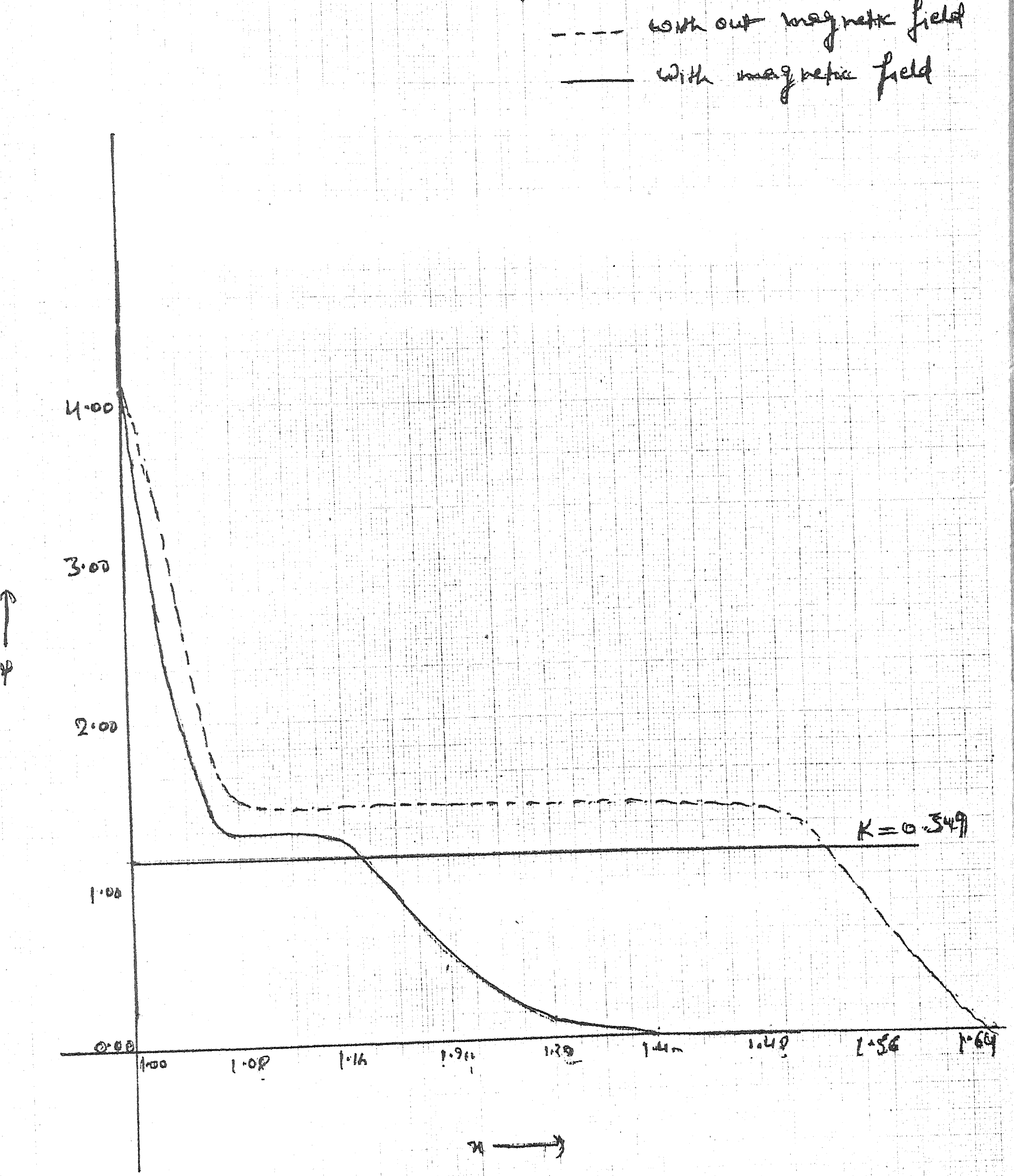
Velocity distribution

Viscous region

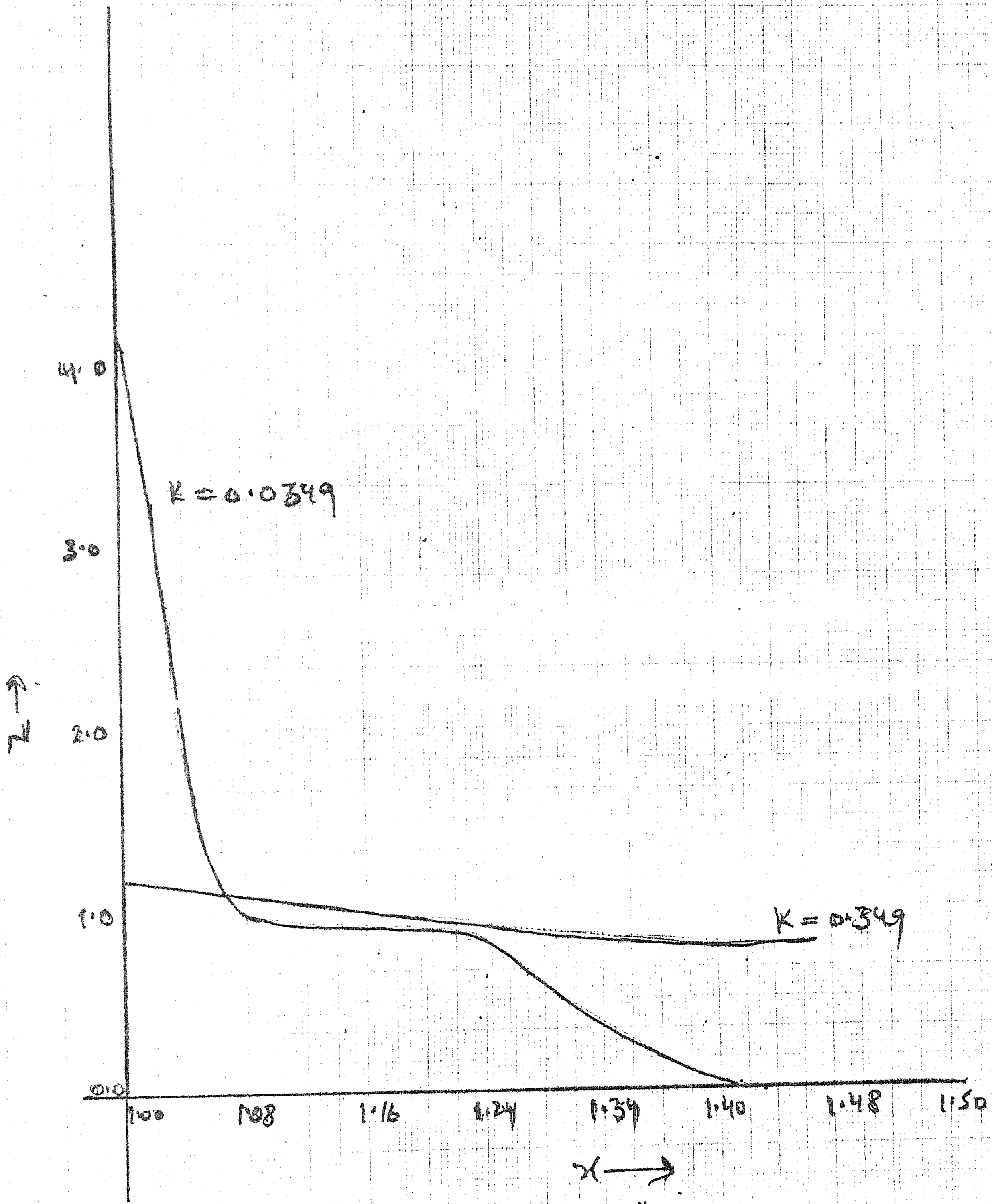


Pressure distribution.

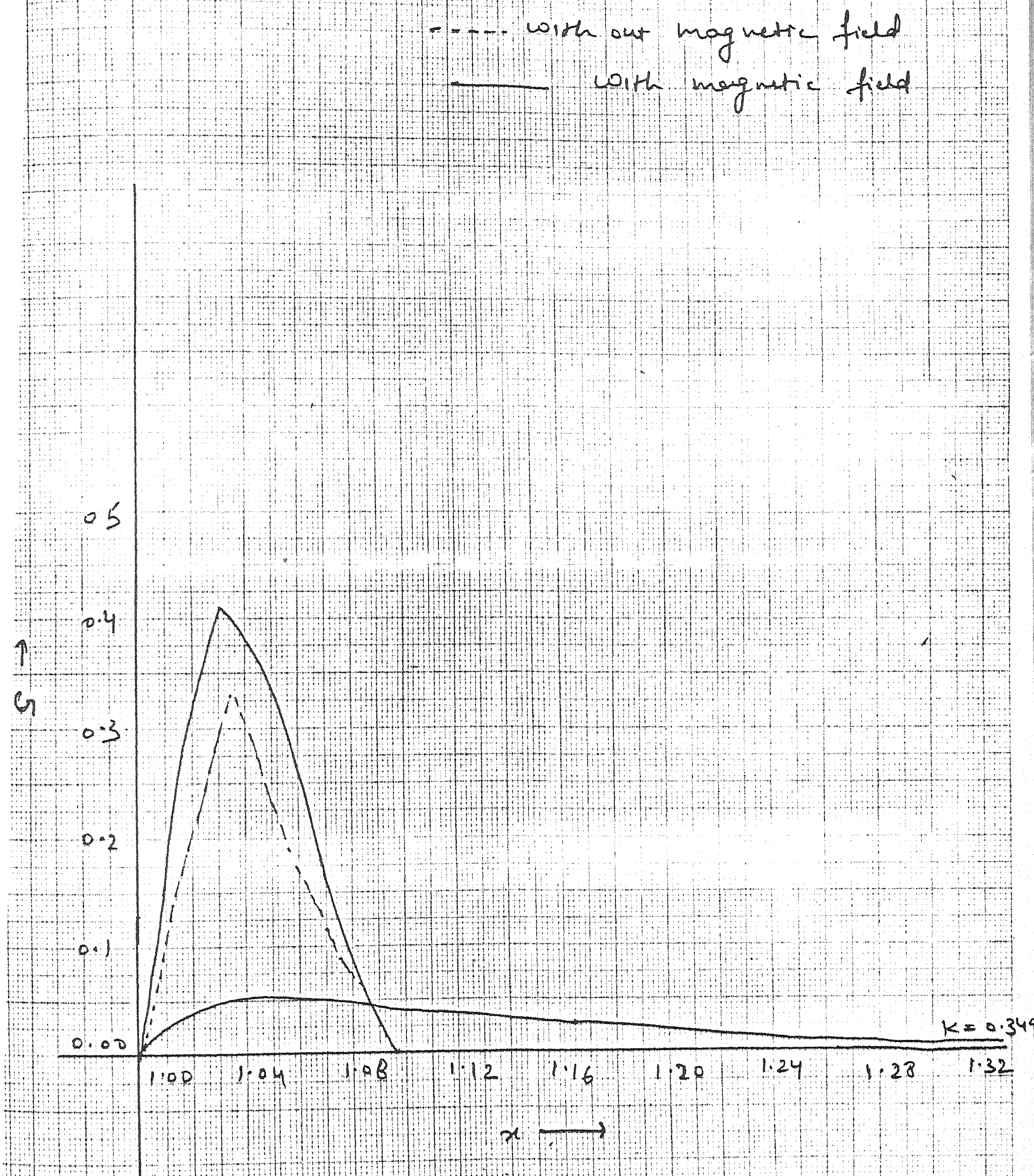
Viscous region



Density distribution.



magnetic field distribution.



Viscosity distribution

SELF-SIMILAR CYLINDRICAL SHOCK WAVES WITH RADIATION HEAT FLUX

INTRODUCTION

The problem of propagation of shock waves in a non-homogeneous medium is of great interest in exploring the effect of explosion in the stars and atmosphere of the earth.

The solution for cylindrically symmetric flow has been obtained numerically by Lin (19), Ray (20) has discussed the problem of point and line explosion and found an exact analytic solution. Analytic solutions in the three cases of plane, cylindrical and spherical flow have been noted by Sakuri (22), Rogers (21) has also studied the similarity solution for these three cases in uniform atmosphere.

In the present paper the problem of explosion along a line in a gas cloud has been discussed. Similarity solutions have been developed describing the propagation of a cylindrical shock in non-uniform atmosphere taking counter gas pressure and radiation heat flux into account. The radiation pressure and radiation energy have been ignored. The gas in the undisturbed field is assumed to be at rest. We also have assumed the gas to be grey and opaque and the shock to be transparent and isothermal.

$$\frac{D\rho}{Dt} + \frac{\rho}{r}\frac{\partial}{\partial r}(ru) = 0 \qquad (3.1)$$

$$\frac{Du}{Dt} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0 \qquad (3.2)$$

$$\frac{DE}{Dt} + p\frac{D}{Dt}\left(\frac{1}{\rho}\right) + \frac{1}{\rho r}\frac{\partial}{\partial r}(rq) = 0 \qquad (3.3)$$

We can written as

$$\frac{\partial}{\partial t}(\log \rho) = \frac{1}{\rho}\frac{\partial \rho}{\partial t} \Rightarrow \frac{\partial \rho}{\partial t} = \rho\frac{\partial(\log \rho)}{\partial t}$$

$$\frac{\partial \rho}{\partial r} = \rho\frac{\partial(\log \rho)}{\partial r}$$

$$\frac{\partial u}{\partial r} = u\frac{\partial(\log u)}{\partial r}$$

Where

$$\eta = r^a t^b$$

$$\frac{\partial \eta}{\partial r} = a r^{a-1} t^b = \frac{a\eta}{r}$$

$$\frac{\partial \eta}{\partial t} = b t^{b-1} r^a = \frac{b\eta}{t}$$

We know that

$$u = \frac{r}{t} V_{(y)}$$

$$\rho = r^{k} t^{\lambda} \Omega_{(y)}$$

$$p = r^{k+2} t^{\lambda-2} P_{(y)}$$

$$q = r^{k+3} t^{\lambda-3} F_{(y)}$$

Differentiate with respect to r and t

$$\rho = r^{k} t^{\lambda} \Omega_{(y)}$$

$$\frac{\partial \rho}{\partial r} = k r^{k-1} t^{\lambda} \Omega_{(y)} + r^{k} t^{\lambda} \frac{\partial \Omega_{(y)}}{\partial y} \frac{\partial y}{\partial r}$$

$$\frac{\partial \rho}{\partial r} = k r^{k-1} t^{\lambda} \Omega_{(y)} + r^{k} t^{\lambda} \frac{a y}{r} \Omega'_{(y)} \quad -[3.4]$$

$$\frac{\partial \rho}{\partial t} = \lambda t^{\lambda-1} r^{k} \Omega_{(y)} + r^{k} t^{\lambda} \frac{\partial \Omega_{(y)}}{\partial y} \frac{\partial y}{\partial t}$$

$$\frac{\partial \rho}{\partial t} = \lambda r^{k} t^{\lambda-1} \Omega_{(y)} + r^{k} t^{\lambda} \Omega'_{(y)} \frac{b y}{t}$$

$$-[3.5]$$

and

$$u = \frac{r}{t} V_{(y)}$$

$$\frac{\partial u}{\partial r} = \frac{1}{t} V_{(y)} + \frac{r}{t} \frac{\partial V_{(y)}}{\partial y} \frac{\partial y}{\partial r}$$

$$\frac{\partial u}{\partial r} = \frac{1}{t} V_{(y)} + \frac{r}{t} \frac{a y}{r} V'_{(y)}$$

$$\frac{\partial u}{\partial r} = \frac{1}{t} v_{(y)} + \frac{r}{t} \frac{ay}{r} v'_{(y)} \qquad (3.6)$$

Put the value from the equation (3.4), (3.5) and (3.6) in (3.1)

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \frac{\rho}{r}\left[u + r \frac{\partial u}{\partial r} \right] = 0$$

$$\Rightarrow \left[\lambda t^{\lambda-1} r^{k} \Omega_{(y)} + r^{k} t^{\lambda} \frac{by}{t} \Omega'_{(y)} \right) + \frac{\rho}{r} \frac{r}{t} v_{(y)}$$
$$+ u\left[k r^{k-1} t^{\lambda} \Omega_{(y)} + r^{k} t^{\lambda} \frac{ay}{r} \right)$$
$$+ r^{k} t^{\lambda} \Omega_{(y)} \left(\frac{1}{t} v_{(y)} + \frac{ay}{t} v'_{(y)} \right] = 0$$

$$\Rightarrow r^{k} t^{\lambda-1} \left[\lambda \Omega_{(y)} + by \Omega'_{(y)} \right] + r^{k} t^{\lambda-1} \Omega_{(y)} v_{(y)}$$
$$+ r^{k} t^{\lambda-1} v_{(y)} \left[k \Omega_{(y)} + ay \Omega'_{(y)} \right]$$
$$+ r^{k} t^{\lambda-1} \Omega_{(y)} \left(v_{(y)} + ay v'_{(y)} \right] = 0$$

$$\Rightarrow \left[\lambda \Omega_{(y)} + by \Omega'_{(y)} \right] + v_{(y)} \left[k \Omega_{(y)} + ay \Omega'_{(y)} \right]$$
$$+ \Omega_{(y)} v_{(y)} + \Omega_{(y)} \left[v_{(y)} + ay v'_{(y)} \right]$$
$$= 0$$

We can put these conditions in this equation

$$K = \omega, \quad \lambda = 0, \quad a = -(4+\omega), \quad b = 2$$

$$\Rightarrow 2y\,\Omega'_{(y)} + \omega\,\vartheta_{(y)}\,\Omega_{(y)} - (4+\omega)\,y\,\Omega'_{(y)}\,\vartheta_{(y)} + \vartheta_{(y)}\,\Omega_{(y)} + \Omega_{(y)}\,\vartheta_{(y)} - (4+\omega)\,y\,\vartheta'_{(y)}\,\Omega_{(y)} = 0$$

$$\Rightarrow \Omega'_{(y)}\left[2y - (4+\omega)\,y\,\vartheta_{(y)}\right] = (4+\omega)\,y\,\vartheta'_{(y)}\,\Omega_{(y)} - (\omega+2)\,\Omega_{(y)}\,\vartheta_{(y)}$$

$$\Rightarrow \frac{\Omega'_{(y)}/A}{\Omega_{(y)}/A} = \frac{(4+\omega)\,y\,\vartheta'_{(y)} - (\omega+2)\,\vartheta_{(y)}}{\left[2y - (4+\omega)\,y\,\vartheta_{(y)}\right]}$$

$$\boxed{\frac{\Omega'_{(y)}/A}{\Omega_{(y)}/A} = \frac{(4+\omega)\,y\,\vartheta'_{(y)} - (\omega+2)\,\vartheta_{(y)}}{y\left[2 - (4+\omega)\,\vartheta_{(y)}\right]}} \qquad \text{—(A)}$$

$$\frac{Du}{Dt} + \frac{1}{\rho}\frac{\partial P}{\partial r} = 0$$

$$\left[\frac{\partial}{\partial t} + u\frac{\partial}{\partial r}\right] u + \frac{1}{\rho}\frac{\partial P}{\partial r} = 0 \qquad \text{— [3.7]}$$

We know that

$$p = r^{k+2}\, t^{\lambda-2}\, P_{(y)}$$

$$\frac{\partial p}{\partial r} = (k+2)\, r^{k+1}\, t^{\lambda-2}\, P_{(y)} + r^{k+2}\, t^{\lambda-2}\, \frac{\partial P_{(y)}}{\partial y}\frac{\partial y}{\partial r}$$

$$\frac{\partial p}{\partial r} = (k+2)\, r^{k+1}\, t^{\lambda-2}\, P_{(y)} + r^{k+2}\, t^{\lambda-2}\, \frac{ay}{r} P'_{(y)} \qquad \text{— [3.8]}$$

$$u = \frac{r}{t} v_{(y)}$$

$$\frac{\partial u}{\partial r} = \frac{1}{t} v_{(y)} + \frac{r}{t}\frac{\partial v_{(y)}}{\partial y}\frac{\partial y}{\partial r}$$

$$\frac{\partial u}{\partial r} = \frac{1}{t} v_{(y)} + \frac{ay}{t} v'_{(y)} \qquad \text{— [3.9]}$$

$$\frac{\partial u}{\partial t} = -\frac{1}{t^2}\, r\, v_{(y)} + \frac{r}{t}\frac{\partial v_{(y)}}{\partial y}\frac{\partial y}{\partial t}$$

$$\frac{\partial u}{\partial t} = -\frac{1}{t^2}\, r\, v_{(y)} + \frac{by}{t}\frac{\partial v_{(y)}}{\partial y} \qquad \text{[3.10]}$$

Put the value from the equation in equation (3.1)

$$\Rightarrow \left[-\frac{1}{t^2}\, r\, V_{(y)} + \frac{b y r}{t^2}\, \frac{\partial V_{(y)}}{\partial y} \right] + \frac{r}{t}\, V_{(y)} \left[\frac{1}{t} V_{(y)} + \frac{a y}{t} V'_{(y)} \right]$$

$$+ \frac{1}{\rho}\left[(k+2)\, r^{k+1}\, t^{\lambda-2}\, P_{(y)} + r^{k+2}\, t^{\lambda-2}\, \frac{a y}{r}\, P'_{(y)} \right] = 0$$

$$\Rightarrow \frac{r}{t^2}\left[-V_{(y)} + b y V'_{(y)} \right] + \frac{r}{t^2}\, V_{(y)} \left[V_{(y)} + a y V'_{(y)} \right]$$

$$+ \frac{r^{k+1}\, t^{\lambda-2}}{r^{k}\, t^{\lambda}\, \Omega_{(y)}} \left[(k+2)\, P_{(y)} + a y P'_{(y)} \right] = 0$$

$$\Rightarrow \left[-V_{(y)} + b y V'_{(y)} \right] + V_{(y)} \left[V_{(y)} + a y V'_{(y)} \right]$$

$$+ \frac{1}{\Omega_{(y)}} \left[(k+2)\, P_{(y)} + a y P'_{(y)} \right] = 0$$

Applying the condition

$$K = \omega, \quad \lambda = 0, \quad a = -(u+\omega), \quad b = 2$$

$$\Rightarrow \left[- \nu_{(\eta)} + 2\eta \, \nu'_{(\eta)} \right] + \nu_{(\eta)} \left[\nu_{(\eta)} - (4+\omega)\eta \, \nu'_{(\eta)} \right]$$

$$+ \frac{1}{\Omega_{(\eta)}} \left[(\omega+2) P_{(\eta)} - (4+\omega)\eta \, P'_{(\eta)} \right) = 0$$

$$\Rightarrow \nu_{(\eta)} \left[\nu_{(\eta)} - 1 \right] + \left[2\eta - (4+\omega)\eta \right] \nu'_{(\eta)}$$

$$+ \frac{1}{\Omega_{(\eta)}} (\omega+2) P_{(\eta)} = (4+\omega)\eta \frac{P'_{(\eta)}}{\Omega_{(\eta)}} \times \frac{P_{(\eta)}}{P_{(\eta)}}$$

$$\Rightarrow \frac{P'_{(\eta)}}{P_{(\eta)}} \frac{P_{(\eta)}}{\Omega_{(\eta)}} = \frac{P_{(\eta)} \, (\omega+2)}{\Omega_{(\eta)} \, (4+\omega)\eta} + \frac{(2\eta - (4+\omega)\eta) \, \nu'_{(\eta)} + \nu_{(\eta)}(\nu_{(\eta)} - 1)}{(4+\omega)\eta}$$

$$\Rightarrow \frac{P'_{(\eta)}/A}{P_{(\eta)}/A} = \frac{(\omega+2)}{(4+\omega)\eta} + \frac{(2\eta - (4+\omega)\eta) \, \nu'_{(\eta)} + \nu_{(\eta)}(\nu_{(\eta)} - 1)}{(4+\omega)\eta} \frac{\Omega_{(\eta)}}{P_{(\eta)}}$$

$$\boxed{\frac{P'_{(\eta)}/A}{P_{(\eta)}/A} = \frac{(\omega+2)}{(4+\omega)\eta} + \frac{(2\eta - (4+\omega)\eta) \, \nu'_{(\eta)} + \nu_{(\eta)}(\nu_{(\eta)} - 1)}{(4+\omega)\eta} \frac{\Omega_{(\eta)}}{P_{(\eta)}}} \qquad \text{(B)}$$

$$\frac{DE}{Dt} + p\frac{D}{Dt}\left(\frac{1}{\rho}\right) + \frac{1}{\rho r}\frac{\partial}{\partial r}(rq) = 0$$

q is a heat flux, t is time and E is a internal energy

$$\Rightarrow \quad \left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial r}\right)E + p\left[\frac{\partial}{\partial t} + u\frac{\partial}{\partial r}\right]\left(\frac{1}{\rho}\right) + \frac{1}{\rho r}\frac{\partial}{\partial r}(rq) = 0$$

$$\Rightarrow \quad \frac{\partial E}{\partial t} + u\frac{\partial E}{\partial r} + p\left[(-1/\rho^2)\frac{\partial \rho}{\partial t}\right] + pu(-1/\rho^2)\frac{\partial \rho}{\partial r} + \frac{1}{\rho r}\left[q + r\frac{\partial q}{\partial r}\right] = 0 \qquad [3.11]$$

for an ideal gas
$$E = \frac{p}{(\gamma-1)\rho}$$

Differntial with respect to r and t we get

$$\frac{\partial E}{\partial t} = \frac{1}{(\gamma-1)\rho}\frac{\partial p}{\partial t} + \frac{p}{(\gamma-1)}\left(-\frac{1}{\rho^2}\right)\frac{\partial \rho}{\partial t} \qquad (3.12)$$

$$\frac{\partial E}{\partial r} = \frac{1}{(\gamma-1)\rho}\frac{\partial p}{\partial r} + \frac{p}{(\gamma-1)}\left(-\frac{1}{\rho^2}\right)\frac{\partial \rho}{\partial r} \qquad (3.13)$$

$$u = \frac{r}{t}\,\nu_{(y)}$$

$$y = r^{a} t^{b}$$

$$\frac{\partial y}{\partial t} = \frac{by}{t} \qquad \text{— [3.14]}$$

$$\frac{\partial y}{\partial r} = \frac{ay}{r} \qquad \text{— [3.15]}$$

$$\rho = r^{k} t^{\lambda}\, \Omega_{(y)}$$

$$\frac{\partial \rho}{\partial t} = \lambda\, t^{\lambda-1} r^{k}\, \Omega_{(y)} + r^{k} t^{\lambda} \frac{by}{t}\, \Omega'_{(y)} \qquad \text{— [3.16]}$$

$$\frac{\partial \rho}{\partial r} = k\, r^{k-1} t^{\lambda}\, \Omega_{(y)} + r^{k} t^{\lambda} \frac{ay}{r}\, \Omega'_{(y)} \qquad \text{— [3.17]}$$

$$p = r^{k+2}\, t^{\lambda-2}\, P_{(y)}$$

$$\frac{\partial p}{\partial t} = (\lambda-2)\, t^{\lambda-3} r^{k+2} P_{(y)} + \frac{by}{t}\, r^{k+2} t^{\lambda-2} P'_{(y)} \qquad \text{— [3.18]}$$

$$\frac{\partial p}{\partial r} = (k+2)\, r^{k+1} t^{\lambda-2} P_{(y)} + \frac{ay}{r}\, r^{k+2} t^{\lambda-2} P'_{(y)} \qquad \text{— [3.19]}$$

$$q = r^{k+3}\, t^{\lambda-3}\, F_{(y)}$$

$$\frac{\partial q}{\partial t} = (\lambda-3)\, t^{\lambda-4} r^{k+3} F_{(y)} + by\, r^{k+3} t^{\lambda-4} F'_{(y)} \qquad \text{— [3.20]}$$

$$\frac{\partial q}{\partial r} = (k+3)\, r^{k+2} t^{\lambda-3} F_{(y)} + ay\, r^{k+2} t^{\lambda-4} F'_{(y)} \qquad \text{— [3.21]}$$

$$u = r/t \; \nu(y)$$

$$\frac{\partial u}{\partial r} = \frac{1}{t} \nu(y) + \frac{r}{t} \frac{a y}{r} \nu'(y)$$

$$\frac{\partial u}{\partial t} = -\frac{1}{t^2} r \; \nu(y) + \frac{b y}{t} \frac{\partial \nu(y)}{\partial y}$$

Put the value from equations (3.12), (3.13), (3.14), (315), (3.16), (3.17), (3.18), (3.19), (3.20), (3.21) in equation (3.1) we get

$$\Rightarrow \left[\frac{1}{(r-1)\rho} \frac{\partial P}{\partial t} - \frac{P}{(r-1)\rho^2} \frac{\partial \rho}{\partial t}\right] + u\left[\frac{1}{(r-1)\rho} \frac{\partial P}{\partial r} - \frac{P}{(r-1)\rho^2} \frac{\partial \rho}{\partial r}\right]$$

$$- \frac{P}{\rho^2}\left[\lambda t^{\lambda-1} r^k \Omega(y) + r^k t^\lambda \frac{b y}{t} \Omega'(y)\right],$$

$$+ \frac{q}{\rho r} + \frac{1}{\rho}\left[(k+3) r^{k+2} t^{\lambda-3} F(y) + \frac{a y}{r} r^{k+3} t^{\lambda-3} F'(y)\right]$$

$$- \frac{P.u}{\rho^2}\left[k \, r^{k-1} t^\lambda \Omega(y) + r^k t^\lambda \frac{a y}{r} \Omega'(y)\right] = 0$$

$$\Longrightarrow \frac{1}{(r-1)f}\left[(\lambda-2)t^{\lambda-3}\,r^{k+2}P_{(y)} + by\, r^{k+2}\,t^{\lambda-3}P'_{(y)}\right]$$

$$-\ \frac{p}{(r-1)f^2}\left[\lambda\, t^{\lambda-1} r^{k}\,\Omega_{(y)} + r^{k}\,t^{\lambda-1}\,by\,\Omega'_{(y)}\right]$$

$$+\ \frac{u}{(r-1)f}\left[(k+2)r^{k+1}\,t^{\lambda-2}P_{(y)} + \frac{ay}{r}\,r^{k+2}t^{\lambda-2}P'_{(y)}\right]$$

$$-\ \frac{pu}{(r-1)f^2}\left[k\,r^{k-1}\,t^{\lambda}\,\Omega_{(y)} + r^{k-1}t^{\lambda}\,ay\,\Omega'_{(y)}\right]$$

$$-\ \frac{p}{f^2}\left[\lambda\ t^{\lambda-1} r^{k}\,\Omega_{(y)} + r^{k}t^{\lambda-1}\,by\,\Omega'_{(y)}\right]$$

$$+\ \frac{q}{fr}$$

$$-\ \frac{pu}{f^2}\left[k\,r^{k-1}t^{\lambda}\,\Omega_{(y)} + r^{k}t^{\lambda}\,\frac{ay}{r}\,\Omega'_{(y)}\right]$$

$$+\ \frac{q}{f}\left[(k+3)\,r^{k+2}\,t^{\lambda-3}F_{(y)} + \frac{ay}{r}\,r^{k+3}t^{\lambda-3}F'_{(y)}\right]$$

$$= 0$$

Put the values of p, ρ, u, q in this equation

$$\Rightarrow \frac{t^{\lambda-3}\, r^{k+2}}{(\gamma-1)\, r^{k} t^{\lambda}\, \Omega(y)}\left[(\lambda-2)\, P(y) + by\, P'(y)\right] + \frac{r^{k+3}\, t^{\lambda-3} F(y)}{r^{k} t^{\lambda}\, \Omega(y)^{2}}$$

$$- \frac{r^{k+2}\, t^{\lambda-2}\, P(y)\, r^{k}\, t^{\lambda-1}}{(\gamma-1)\, (r^{k} t^{\lambda}\, \Omega(y))^{2}}\left[\lambda\, \Omega(y) + by\, \Omega'(y)\right]$$

$$+ \frac{\frac{r}{t}\, V(y)\, r^{k+1}\, t^{\lambda-2}}{(\gamma-1)\, (r^{k} t^{\lambda}\, \Omega(y))}\left[(k+2) P(y) + ay\, P'(y)\right]$$

$$- \frac{r^{k+3}\, t^{\lambda-3}\, F(y)\, \frac{r}{t}\, V(y)\, r^{k-1}\, t^{\lambda}}{(\gamma-1)\, (r^{k} t^{\lambda})^{2}\, \Omega(y)^{2}}\left[k\, \Omega(y) + ay\, \Omega'(y)\right]$$

$$- \frac{r^{k+2}\, t^{\lambda-2}\, P(y)\, r^{k}\, t^{\lambda-1}}{(r^{k} t^{\lambda}\, \Omega(y))^{2}}\left[\lambda\, \Omega(y) + by\, \Omega'(y)\right]$$

$$- \frac{r^{k+2}\, t^{\lambda-2}\, P(y)\, \frac{r}{t}\, V(y)\, r^{k-1}\, t^{\lambda}}{(r^{k} t^{\lambda}\, \Omega(y))^{2}}\left[k\, \Omega(y) + ay\, \Omega'(y)\right]$$

$$+ \frac{r^{k+3}\, t^{\lambda-3} F(y)\, r^{k+2}\, t^{\lambda-3}}{r^{k} t^{\lambda}\, \Omega(y)}\left[(k+3)\, F(y) + ay\, F'(y)\right]$$

$$= 0$$

$$\Rightarrow \quad \frac{1}{(r-1)\,\Omega(y)}\left[(\lambda-2)\,P(y) + by\,P(y)'\right] + \frac{F(y)}{\Omega(y)}$$

$$- \frac{P(y)}{(r-1)\,\Omega(y)^2}\left[\lambda\,\Omega(y) + by\,\Omega(y)'\right]$$

$$- \frac{P(y)}{(\Omega(y))^2}\left[\lambda\,\Omega(y) + by\,\Omega(y)'\right] + \frac{\vartheta(y)}{(r-1)\,\Omega(y)}\left[(k+2)\,P(y) + ay\,P(y)'\right]$$

$$- \frac{P(y)\,\vartheta(y)}{(r-1)\,(\Omega(y))^2}\left[k\,\Omega(y) + ay\,\Omega(y)'\right]$$

$$- \frac{P(y)\,\vartheta(y)}{(\Omega(y))^2}\left[k\,\Omega(y) + ay\,\Omega(y)'\right] + \frac{1}{\Omega(y)}\left[(k+3)\,F(y) + ay\,F(y)'\right]$$

$$= 0$$

$$\Rightarrow \quad \frac{P(y)'}{(r-1)\,\Omega(y)}\left[by + ay\,\vartheta(y)\right] + \frac{P(y)}{(r-1)\,\Omega(y)}\left[(\lambda-2) + (k+2)\,\vartheta(y)\right]$$

$$- \frac{F(y)}{\Omega(y)} + \frac{P(y)\,\Omega(y)'}{(\Omega(y))^2}\left[\frac{-by}{(r-1)} - ay\,\frac{\vartheta(y)}{(r-1)} - by - ay\,\vartheta(y)\right]$$

$$- \frac{P(y)\,\Omega(y)}{(\Omega(y))^2}\left[\frac{-\lambda}{(r-1)} - \frac{k\,\vartheta(y)}{(r-1)} - \lambda - k\,\vartheta(y)\right]$$

$$+ \frac{F(y)}{\Omega(y)}\left[(k+3) + ay\,\frac{F(y)'}{F(y)}\right]$$

$$= 0$$

Put these conditions

$$k = \omega, \quad \lambda = 0, \quad b = 2, \quad a = -(4+\omega)$$

$$\Rightarrow \frac{P'_{(y)}}{(r-1)\,\Omega_{(y)}}\left[2y - (4+\omega)\,y\,\vartheta_{(y)}\right] + \frac{P_{(y)}}{(r-1)\,\Omega_{(y)}}\left[-2 + (\omega+2)\,\vartheta_{(y)}\right]$$

$$- \frac{F_{(y)}}{\Omega_{(y)}} - \frac{P_{(y)}\,\Omega_{(y)}}{(\Omega_{(y)})^2}\left[-0 - \frac{\omega\,\vartheta_{(y)}}{(r-1)} - 0 - \omega\,\vartheta_{(y)}\right]$$

$$+ \frac{P_{(y)}\,\Omega'_{(y)}}{(\Omega_{(y)})^2}\left[\frac{-2y}{(r-1)} - \frac{(4+\omega)\,y\,\vartheta_{(y)}}{(r-1)} - 2y - (4+\omega)\,y\,\vartheta_{(y)}\right]$$

$$+ \frac{F_{(y)}}{\Omega_{(y)}}\left[(\omega+3) - (4+\omega)\,y\,\frac{F'_{(y)}}{F_{(y)}}\right] = 0$$

$$\Longrightarrow \frac{P'_{(y)}\,y}{(r-1)\,\Omega_{(y)}}\left[2 - (4+\omega)\,\vartheta_{(y)}\right] + \frac{P_{(y)}}{\Omega_{(y)}}\left[\frac{(\omega+2)\,\vartheta_{(y)} - 2 - \omega r\,\vartheta_{(y)}}{(r-1)}\right]$$

$$+ \frac{F_{(y)}}{\Omega_{(y)}} + \frac{P_{(y)}\,\Omega'_{(y)}}{(\Omega_{(y)})^2}\left[-2y\left(\frac{r}{(r-1)}\right) + (4+\omega)\,y\,\vartheta_{(y)}\left(\frac{r}{(r-1)}\right)\right]$$

$$+ \frac{F_{(y)}}{\Omega_{(y)}}\left[(\omega+3) - (4+\omega)\,y\,\frac{F'_{(y)}}{F_{(y)}}\right] = 0$$

$$\Rightarrow \frac{P'_{(y)}\, y}{(r-1)\,\Omega_{(y)}}\left[2-(4+\omega)\,\vartheta_{(y)}\right] + \frac{P_{(y)}}{\Omega_{(y)}}\left[\frac{(\omega+2)\,\vartheta_{(y)} - 2 - \omega r\, \vartheta'_{(y)}}{(r-1)}\right]$$

$$+ \frac{P_{(y)}}{\Omega_{(y)}}\left[\frac{y\,(4+\omega)\,\vartheta'_{(y)} - (2+\omega)\,\vartheta_{(y)}}{y\,(2-\cancel{(4+\omega)}\,\vartheta_{(y)})}\right]\left[\frac{-y\, r\left[2-\cancel{(4+\omega)\,\vartheta_{(y)}}\right]}{(r-1)}\right]$$

$$+ \frac{F_{(y)}}{\Omega_{(y)}}\left[(\omega+4)\right] = (4+\omega)\, y\, \frac{F'_{(y)}}{F_{(y)}}\, \frac{F_{(y)}}{\Omega_{(y)}}$$

$$\Rightarrow \frac{F'_{(y)}/A}{F_{(y)}/A} = \frac{P'_{(y)}}{F_{(y)}}\left[\frac{2-(4+\omega)\,\vartheta_{(y)}}{(4+\omega)(r-1)}\right] + \frac{1}{y}$$

$$+ \frac{P_{(y)}}{F_{(y)}}\left[\frac{(\omega+2)\,\vartheta_{(y)} - 2 - \omega r\,\vartheta_{(y)} - r\, y\,(4+\omega)\,\vartheta'_{(y)} + (2+\omega)\,\vartheta_{(y)}}{y\,(r-1)\,(4+\omega)}\right]$$

$$\frac{F'_{(y)}/A}{F_{(y)}/A} = \frac{P'_{(y)}}{F_{(y)}}\left[\frac{2-(4+\omega)\,\vartheta_{(y)}}{(4+\omega)(r-1)}\right] + \frac{1}{y}$$

$$+ \frac{P_{(y)}}{F_{(y)}}\left[\frac{(\omega+2r+2)\,\vartheta_{(y)} - r\left[(4+\omega)\, y\, \vartheta'_{(y)} - 2\right]}{y\,(r-1)(4+\omega)}\right]$$

$$\longrightarrow (c)$$

For an ideal gas

$$P = \sigma \rho T$$

$$T = \frac{P}{\sigma \rho}$$

Taking Rosseland's diffusion approximation we have

$$q = \frac{-c\mu}{3} \frac{\partial}{\partial r} \left(\sigma T^4 \right)$$

Put the value of T

$$q = \frac{-c\mu}{3} \frac{\partial}{\partial r} \left[\sigma \frac{P^4}{T^4 \rho^4} \right]$$

$$q = \frac{-c\mu\sigma}{3T^4} \frac{\partial}{\partial r} \left(\frac{P^4}{\rho^4} \right)$$

We know that $\mu = \mu_0 \rho^\alpha T^\beta$

Substitute the value of μ

$$q = \frac{-c\mu_0 \rho^\alpha T^\beta \sigma}{3T^4} \frac{\partial}{\partial r} \left(\frac{P^4}{\rho^4} \right)$$

$$q = \frac{-c\mu_0 \sigma \rho^\alpha P^\beta}{3T^{4+\beta} \rho^\beta} \frac{\partial}{\partial r} \left(\frac{P^4}{\rho^4} \right)$$

Evalute the value of $\frac{\partial}{\partial r}\left(\frac{P^4}{f^4}\right)$

$$\frac{\partial}{\partial r}\left(\frac{P^4}{f^4}\right) = \frac{\partial}{\partial r}\left[\frac{(r^{k+2}\, t^{1-2}\, P_{(\eta)})^4}{(r^k\, t^1\, \Omega_{(\eta)})^4}\right]$$

$$\frac{\partial}{\partial r}\left(\frac{P^4}{f^4}\right) = \frac{\partial}{\partial r}\left[\frac{r^8}{t^8}\;\frac{(P_{(\eta)})^4}{(\Omega_{(\eta)})^4}\right]$$

$$\Rightarrow \frac{\partial}{\partial r}\left(\frac{P^4}{f^4}\right)$$

$$= \frac{1}{t^8}\left[\frac{(\Omega_{(\eta)})^4\left[8r^7 (P_{(\eta)})^4 + r^8\, 4(P_{(\eta)})^3 P'_{(\eta)} \frac{\partial\eta}{\partial r}\right] - r^8 P_{(\eta)}^4\, 4\, \Omega_{(\eta)}^3\, \Omega'_{(\eta)} \frac{\partial\eta}{\partial r}}{(\Omega_{(\eta)})^8}\right]$$

$$\Rightarrow \frac{\partial}{\partial r}\left(\frac{P^4}{f^4}\right) = \frac{4}{t^8}\left[\frac{\Omega_{(\eta)}^4 (P_{(\eta)})^3\left[2r^7 P_{(\eta)} + r^8 \frac{a\eta}{r} P'_{(\eta)}\right] - r^8 \frac{a\eta}{r} P_{(\eta)}^4\, \Omega_{(\eta)}^3\, \Omega'_{(\eta)}}{(\Omega_{(\eta)})^8}\right]$$

$$\Rightarrow \frac{\partial}{\partial r}\left(\frac{P^4}{\rho^4}\right)$$

$$= \frac{4r^7}{t^8} P_{(y)}{}^3 \, \Omega_{(y)}{}^3 \times$$

$$\times \left[\frac{2 P_{(y)} \Omega_{(y)} + ay \left[\Omega_{(y)} P'_{(y)} - P_{(y)} \Omega'_{(y)} \right]}{(\Omega_{(y)})^8} \right]$$

$$\Rightarrow \frac{\partial}{\partial r}\left(\frac{P^4}{\rho^4}\right)$$

$$= \frac{4r^7}{t^8} \frac{P_{(y)}{}^3}{(\Omega_{(y)})^5} \left[2 P_{(y)} \Omega_{(y)} + ay \left[\Omega_{(y)} P'_{(y)} - P_{(y)} \Omega'_{(y)} \right] \right]$$

$$\Rightarrow \frac{\partial}{\partial r}\left(\frac{P^4}{\rho^4}\right)$$

$$= \frac{4r^7}{t^8} \frac{P_{(y)}{}^3}{(\Omega_{(y)})^5} P_{(y)} \Omega_{(y)} \left[2 + ay \left[\frac{P'_{(y)}}{P_{(y)}} - \frac{\Omega'_{(y)}}{\Omega_{(y)}} \right] \right]$$

Put the value of $\frac{\partial}{\partial r}\left(\frac{p^4}{\rho^4}\right)$ in equation (3.1)

$$q = \frac{-c\,h_0\,\sigma\,\rho^{(\alpha-\beta)}\,p^{\beta}}{3\,T^{4+\beta}}\;\frac{4r^7}{t^8}\;\frac{(P_{(y)})^4}{(\Omega_{(y)})^4}\;\times$$

$$\times\left[2 + ay\left[\frac{P'_{(y)}}{P_{(y)}} - \frac{\Omega'_{(y)}}{\Omega_{(y)}}\right]\right]$$

Substitute the value of q, ρ , p

$$\Rightarrow\quad r^{k+3}\,t^{1-3}\,F_{(y)} = \frac{-4c\,h_0\,\sigma}{3\,T^{4+\beta}}\,(r^{k}\,t^{1}\,\Omega_{(y)})^{\alpha-\beta}$$

$$\times\,(r^{k+2}\,t^{1-2}\,P_{(y)})^{\beta}\;\times$$

$$\times\;\frac{r^7}{t^8}\;\frac{(P_{(y)})^3}{(\Omega_{(y)})^5}\left[2\,\Omega_{(y)}\,P_{(y)} + ay\left[P'_{(y)}\,\Omega_{(y)} - \Omega'_{(y)}\,P_{(y)}\right]\right]$$

$$\Rightarrow\quad r^{k+3}\,t^{1-3}\,F_{(y)} = \frac{-4c\,h_0\,\sigma}{3T^{4+\beta}}\;r^{k\alpha+2\beta+7}\;t^{1\alpha-2\beta-8}\;\times$$

$$\times\;\frac{(P_{(y)})^{\beta+3}}{(\Omega_{(y)})^{5-\alpha-\beta}}\left[2\,P_{(y)}\,\Omega_{(y)} + ay\left[P'_{(y)}\,\Omega'_{(y)} - \Omega'_{(y)}\,P_{(y)}\right]\right]$$

Comparing both side

$$r^{k+3}\, t^{\lambda-3} = r^{k\alpha+2\beta+7}\, t^{\lambda\alpha-2\beta-8}$$

$$k+3 = k\alpha+2\beta+7 \qquad \text{—— (3.22)}$$

$$\lambda-3 = \lambda\alpha-2\beta-8 \qquad \text{—— (3.23)}$$

$$k+\lambda = k\alpha+\lambda\alpha-1$$

$$\lambda(1-\alpha) = k(\alpha-1)-1$$

Substitute the value of λ and k

$$\lambda = 0 \quad , \quad k = \omega$$

$$0 = \omega\alpha - \omega - 1$$

$$\omega\alpha = (\omega+1)$$

$$\alpha = (\omega+1)/\omega$$

Substitute the value of k and λ in (3.23)

$$0-3 = 0-2\beta-8$$

$$2\beta = -5$$

$$\beta = -5/2$$

Substitute the value of α and β in equation

$$F_{(y)} = \frac{-4\, c\, h_0\, \sigma}{3\, T^{4-5/2}} \; \frac{(P_{(y)})^{1/2}}{(\Omega_{(y)})^{5/2-\alpha}} \times$$

$$\times \left[2\, P_{(y)}\, \Omega_{(y)} + a y \left[P'_{(y)}\, \Omega_{(y)} - \Omega'_{(y)}\, P_{(y)} \right] \right]$$

divided both side by $P_{(y)}\, \Omega_{(y)}$

$$\Rightarrow \quad \frac{F_{(y)}}{P_{(y)}\, \Omega_{(y)}} = \frac{-4\, c\, h_0\, \sigma}{3\, T^{3/2}} \; \frac{(P_{(y)})^{1/2}}{(\Omega_{(y)})^{5/2-\alpha}} \times$$

$$\times \left[2 + a y \left[\frac{P'_{(y)}}{P_{(y)}} - \frac{\Omega'_{(y)}}{\Omega_{(y)}} \right] \right]$$

$$\Rightarrow \quad \frac{F_{(y)}}{P_{(y)}} = \frac{-4\, c\, h_0\, \sigma}{3\, T^{3/2}} \; \frac{(P_{(y)})^{1/2}}{(\Omega_{(y)})^{5/2-\alpha}\, (\Omega_{(y)})^{-1}} \times$$

$$\times \left[2 + a y \left[\frac{P'_{(y)}}{P_{(y)}} - \frac{\Omega'_{(y)}}{\Omega_{(y)}} \right] \right]$$

$$\Rightarrow \frac{F_{(y)}/A}{P_{(y)}/A} = \frac{-4ch_0\sigma}{3T^{3/2}} \frac{P_{(y)}^{1/2} / A^{3/2-\alpha}}{(\Omega_{(y)})^{3/2-\alpha} / A^{3/2-\alpha}} \times \left[2 + ay\left[\frac{P_{(y)}'}{P_{(y)}} - \frac{\Omega_{(y)}'}{\Omega_{(y)}}\right]\right]$$

$$\Rightarrow \frac{F_{(y)}/A}{P_{(y)}/A} = \frac{-4ch_0\sigma}{3T^{3/2}A^{(1-\alpha)}} \frac{(P_{(y)})^{1/2} / A^{1/2}}{(\Omega_{(y)})^{3/2-\alpha} / A^{3/2-\alpha}} \times \left[2 + ay\left[\frac{P_{(y)}'}{P_{(y)}} - \frac{\Omega_{(y)}'}{\Omega_{(y)}}\right]\right]$$

As $\left[A^{3/2-\alpha} = A^{1-\alpha} A^{1/2}\right]$

$$\Rightarrow \frac{F_{(y)}/A}{P_{(y)}/A} = \frac{-4ch_0\sigma}{3T^{3/2}A^{(1-\alpha)}} \frac{(P_{(y)}/A)^{1/2}}{(\Omega_{(y)}/A)^{3/2-\alpha}} \times \left\{2 + ay\left[\frac{P_{(y)}'}{P_{(y)}} - \frac{\Omega_{(y)}'}{\Omega_{(y)}}\right]\right]$$

$$\frac{F_{(y)}/A}{P_{(y)}/A} = \frac{-4ch_0\sigma}{3T^{3/2}A^{(1-\alpha)}} \frac{(P_{(y)}/A)^{1/2}}{(\Omega_{(y)}/A)^{3/2-\alpha}} \times \left[2 - (4+\omega)y\left[\frac{P_{(y)}'}{P_{(y)}} - \frac{\Omega_{(y)}'}{\Omega_{(y)}}\right]\right] \quad \text{——(I)}$$

Equation (D) in substitute the value of $\frac{P'_{(y)}}{P_{(y)}}$ and $\frac{\Omega'_{(y)}}{\Omega_{(y)}}$

$$N = \frac{-4c\sigma\mu_0}{3\tau^{3/2}A^{(1-\alpha)}}$$

where

$$N = \frac{4c\sigma\mu_0}{3\tau^{3/2}A^{(1-\alpha)}} = \text{A non dimensional radiation parameter}$$

$$\Rightarrow \frac{f_{(y)}/A}{P_{(y)}/A} = -N\frac{(P_{(y)}/A)^{1/2}}{(\Omega_{(y)}/A)^{3/2-\alpha}}\Big[2-(4+\omega)\eta\Big\{ \rightarrow$$

$$\rightarrow \Big[\frac{\Omega_{(y)}/A}{P_{(y)}/A}\Big[\frac{\eta\nu'_{(y)}\{2-(4+\omega)\nu_{(y)}\}+\nu_{(y)}(\nu_{(y)}-1)}{\eta(4+\omega)}\Big]$$

$$+\frac{(2+\omega)}{\eta(4+\omega)} - \Big[\frac{\eta(4+\omega)\nu'_{(y)}-(2+\omega)\nu_{(y)}}{\eta(2-(4+\omega))\nu_{(y)}}\Big]$$

$$\Rightarrow \frac{1}{N}\frac{f_{(y)}/A}{P_{(y)}/A}\times\frac{(\Omega_{(y)}/A)^{3/2-\alpha}}{(P_{(y)}/A)^{1/2}}$$

$$= -2+(4+\omega)\eta\frac{(2+\omega)}{\eta(4+\omega)}$$

$$+(4+\omega)\eta\Big\{\frac{\Omega_{(y)}/A}{P_{(y)}/A}\Big[\frac{\eta\nu'_{(y)}\{2-(4+\omega)\nu_{(y)}\}+\nu_{(y)}(\nu_{(y)}-1)}{\eta(4+\omega)}\Big\}$$

$$-\eta(4+\omega)\Big[\frac{\eta(4+\omega)\nu'_{(y)}-(2+\omega)\nu_{(y)}}{\eta\{(2-(4+\omega))\nu_{(y)}\}}\Big\}$$

$$\Rightarrow \quad \frac{1}{N}\,\frac{(F_{(y)}/A)\,(\Omega_{(y)}/A)^{3/2-\alpha}}{(P_{(y)}/A)^{3/2}}$$

$$= \omega - (4+\omega)\left[\frac{y(4+\omega)\nu'_{(y)} - (2+\omega)\nu_{(y)}}{(2-(4+\omega)\nu_{(y)})}\right]$$

$$+ \frac{(\Omega_{(y)}/A)}{(P_{(y)}/A)}\Big[y\,\nu'_{(y)}\left\{2-(4+\omega)\nu_{(y)}\right\} + \nu_{(y)}(\nu_{(y)}-1)\Big]$$

$$\Rightarrow \quad \frac{1}{N}\,\frac{(F_{(y)}/A)\,(\Omega_{(y)}/A)^{3/2-\alpha}}{(P_{(y)}/A)^{3/2}}\left[2-(4+\omega)\nu_{(y)}\right]$$

$$= \omega\left[2-(4+\omega)\nu_{(y)}\right] - \left[y(4+\omega)^2\nu'_{(y)} - (4+\omega)(2+\omega)\nu_{(y)}\right]$$

$$+ \frac{(\Omega_{(y)}/A)}{(P_{(y)}/A)}\Big[y\,\nu'_{(y)}\left\{2-(4+\omega)\nu_{(y)}\right\} + \nu_{(y)}(\nu_{(y)}-1)\Big]\left[2-(4+\omega)\nu_{(y)}\right]$$

$$\Rightarrow \frac{1}{N}\frac{(F_{(y)}/A)(\Omega_{(y)}/A)^{3/2-\alpha}}{(P_{(y)}/A)^{3/2}}\left[2-(4+\omega)\nu_{(y)}\right] = -\eta(4+\omega)^2\nu_{(y)}'$$

$$+\omega(2-(4+\omega)\nu_{(y)}) + \frac{\Omega_{(y)}/A}{P_{(y)}/A}\eta\nu_{(y)}'\left[2-(4+\omega)\nu_{(y)}\right]^2$$

$$+\frac{\Omega_{(y)}/A}{P_{(y)}/A}\nu_{(y)}'(\nu_{(y)}-1)\left[2-(4+\omega)\nu_{(y)}\right] + (4+\omega)(2+\omega)\nu_{(y)}\,\nu_{(y)})$$

Multiply by $\frac{P_{(y)}/A}{\Omega_{(y)}/A}$ in the whole expression we obtain

$$\Rightarrow \frac{1}{N}\frac{(F_{(y)}/A)(\Omega_{(y)}/A)^{3/2-\alpha}}{(P_{(y)}/A)^{3/2}}\left[2-(4+\omega)\nu_{(y)}\right]\frac{P_{(y)}/A}{\Omega_{(y)}/A}$$

$$= \frac{P_{(y)}/A}{\Omega_{(y)}/A}\left[2\omega-(4+\omega)\omega\nu_{(y)}+(4+\omega)(2+\omega)\nu_{(y)}\right]$$

$$+\eta\nu_{(y)}'\left[2-(4+\omega)\nu_{(y)}\right]^2+\nu_{(y)}(\nu_{(y)}-1)\times$$
$$\times\left[2-(4+\omega)\nu_{(y)}\right]$$

$$-\frac{P_{(y)}/A}{\Omega_{(y)}/A}\eta\left[4+\omega\right]^2\nu_{(y)}'$$

$$\Rightarrow \frac{1}{N}\frac{(F_{(y)}/A)(\Omega_{(y)}/A)^{1/2-\alpha}}{(P_{(y)}/A)^{1/2}}\left[2-(4+\omega)\nu_{(y)}\right]$$

$$= \frac{P_{(y)}/A}{\Omega_{(y)}/A}\cdot\left[2\omega-4\omega\nu_{(y)}-\omega^2\nu_{(y)}+8\nu_{(y)}\right.$$
$$\left.+\omega^2\nu_{(y)}+4\omega\nu_{(y)}+2\omega\nu_{(y)}\right]$$

$$+\eta\nu_{(y)}'\left\{\left[(2-(4+\omega)\nu_{(y)})\right]^2-\left(\frac{P_{(y)}/A}{\Omega_{(y)}/A}\right)(4+\omega)^2\right\}$$

$$+\nu_{(y)}(\nu_{(y)}-1)\left[2-(4+\omega)\nu_{(y)}\right]$$

$$\Rightarrow \frac{1}{N} \frac{(F_{(y)}/A)(\Omega_{(y)}/A)^{\gamma_2-\alpha}}{(P_{(y)}/A)^{\gamma_2}}\left[2-(u+\omega)\nu_{(y)}\right] - 2\frac{P_{(y)}/A}{\Omega_{(y)}/A}\left\{\omega+(u+\omega)\times\nu_{(y)}\right\}$$

$$= \nu_{(y)}(\nu_{(y)}-1)\left[2-(u+\omega)\nu_{(y)}\right]$$
$$+ \nu_{(y)}'\left[\eta\left\{2-(u+\omega)\nu_{(y)}\right\}^2 - \frac{P_{(y)}/A}{\Omega_{(y)}/A}(u+\omega)^2\right]$$

$$\Rightarrow \frac{1}{N} \frac{(F_{(y)}/A)(\Omega_{(y)}/A)^{\gamma_2-\alpha}}{(P_{(y)}/A)^{\gamma_2}}\left(2-(u+\omega)\nu_{(y)}\right]$$
$$- 2\frac{P_{(y)}/A}{\Omega_{(y)}/A}\left[\omega+(u+\omega)\nu_{(y)}\right)$$

$$= \nu_{(y)}(\nu_{(y)}-1)\left[2-(u+\omega)\nu_{(y)}\right]$$
$$+ \nu_{(y)}'\left[\eta\left[(2-(u+\omega)\nu_{(y)}\right]^2 - \frac{P_{(y)}/A}{\Omega_{(y)}/A}\times(u+\omega)^2\right]$$

$$\Rightarrow \frac{1}{N} \frac{(F_{(y)}/A)(\Omega_{(y)}/A)^{\gamma_2-\alpha}}{(P_{(y)}/A)^{\gamma_2}}\left[2-(u+\omega)\nu_{(y)}\right] - 2\frac{P_{(y)}/A}{\Omega_{(y)}/A}\nu_{(y)}'$$

$$= \frac{\left[(u+\omega)\nu_{(y)}+2\omega\right] - \left[2-(u+\omega)\nu_{(y)}\right]\left[\nu_{(y)}(\nu_{(y)}-1)\right]}{\eta\left\{(2-(u+\omega)\nu_{(y)})\right\}^2 - (u+\omega)^2\,\dfrac{P_{(y)}/A}{\Omega_{(y)}/A}}$$

We have $\eta_0 = \text{Constant}$

$$V = -\frac{b}{a} R/t$$

μ and α, β are constant disturbance is heated by an isothermal shock and the conditions are

$$P_2 - P_1 = m_s u_2$$

$$P_2 = m_s u_2 + \frac{P_1 \gamma \rho_1}{\gamma \rho_1}$$

$$\frac{P_2}{m_s} - \frac{V^2}{\gamma M^2 m_s} \rho_1 = u_2$$

$$\text{As} \quad M^2 = \frac{V^2 \rho_1}{\gamma p_1}$$

Substitute the value of $m_s = \rho_2 (V - u_2) = \rho_1 V = m_s$

$$\frac{\gamma}{\gamma} \frac{P_2}{\rho_2 (V - u_2)} - \frac{V^2}{\gamma M^2 \rho_1 V} \rho_1 = u_2$$

$$\frac{\gamma P_2}{\gamma \rho_2 (V - u_2)} - \frac{V}{\gamma M^2} = u_2$$

$$u_2 = \frac{v^2}{\gamma M^2 (v - u_2)} - \frac{v}{\gamma M^2}$$

$$u_2 = \frac{v^2 - v^2 + u_2 v}{\gamma M^2 (v - u_2)}$$

$$\cancel{u_2} = \cancel{u_2} v \,/\, \gamma M^2 (v - u_2)$$

$$v = \gamma M^2 (v - u_2)$$

$$\Longrightarrow \quad v = \gamma M^2 v - \gamma M^2 u_2$$

$$\Longrightarrow \quad \gamma M^2 u_2 = v (\gamma M^2 - 1)$$

$$u_2 = v \left[1 - \frac{1}{\gamma M^2} \right]$$

$$\frac{R}{t} v(y_0) = -\frac{b}{a} \frac{R}{t} \left[1 - \frac{1}{\gamma M^2} \right]$$

$$\text{As} \quad \left\{ v = -\frac{b}{a} \frac{R}{t} \right\}$$

$$v(y_0) = \frac{2}{(u + \omega)} \left[1 - \frac{1}{\gamma M^2} \right]$$

Now

$$l_2 (v - u_2) = l_1 v$$

$$u_2 = v \left[1 - \frac{1}{rM^2} \right]$$

$$l_1 v = l_2 \left[v - v \left[1 - \frac{1}{rM^2} \right] \right]$$

$$l_1 \cancel{v} = \cancel{v}\, l_2 \left[\cancel{v} - \cancel{v} + \frac{1}{rM^2} \right]$$

$$l_2 \frac{1}{rM^2} = l_1$$

$$\frac{l_2}{l_1} = rM^2$$

$$\frac{R^{K+\lambda}\, \Omega(y_0)}{A R^{\omega}} = rM^2$$

$K = \omega$ and $\lambda = 0$ we get

$$\frac{R^{\omega}\, \Omega(y_0)}{A R^{(\omega)}} = rM^2$$

Ok,

$$\boxed{\frac{\Omega(y_0)}{A} = \gamma M^2}$$

and,

$$E_2 = \frac{p_2}{(\gamma-1)\rho_2}$$

$$E_1 = \frac{p_1}{(\gamma-1)\rho_1}$$

Also

$$T_2 = T_1$$

$$\Rightarrow \frac{p_2}{\rho_2} = \frac{p_1}{\rho_1}$$

$$\Rightarrow E_2 + \frac{p_2}{\rho_2} + \frac{1}{2}(v-u_2)^2 - \frac{q}{m_s}$$

$$= E_1 + \frac{p_1}{\rho_1} + \frac{1}{2}v^2$$

$$\Rightarrow \frac{p_2}{(\gamma-1)\rho_2} + \frac{p_2}{\rho_2} + \frac{1}{2}(v-u_2)^2 - \frac{q_2}{m_s}$$

$$= \frac{p_1}{(\gamma-1)\rho_1} + \frac{p_1}{\rho_1} + \frac{1}{2}v^2$$

or,

$$\Rightarrow \frac{p_1}{(r-1)\rho_1} + \frac{p_1}{\rho_1} + \frac{1}{2}(v-u_2)^2 - \frac{q_2}{m_s}$$

$$= \frac{p_1}{(r-1)\rho_1} + \frac{p_1}{\rho_1} + \frac{1}{2}v^2$$

$$\Rightarrow \frac{1}{2}(v-u_2)^2 - \frac{q_2}{m_s} = \frac{1}{2}v^2$$

$$\Rightarrow \frac{1}{2}v^2 + \frac{1}{2}u_2^2 - vu_2 - \frac{q_2}{m_s} = \frac{1}{2}v^2$$

$$\Rightarrow \frac{1}{2}u_2^2 - vu_2 - \frac{q^2}{m_s} = 0$$

$$\Rightarrow \frac{1}{2}u_2^2 - vu_2 - \frac{q_2}{\rho_1 v} = 0$$

$$\Rightarrow \frac{1}{2}u_2^2 - vu_2 - \frac{q_2}{\rho_1 v} = 0$$

$$\Rightarrow \frac{1}{2}v^2\left[1 - \frac{1}{rM^2}\right]^2 - v^2\left[1 - \frac{1}{rM^2}\right]$$

$$= \frac{q^2}{\rho_1 v}$$

$$\Rightarrow \quad v^2\left[1-\frac{1}{rM^2}\right]\left[\frac{1}{2}\left\{1-\frac{1}{rM^2}\right\}-1\right] = \frac{q_2}{\rho_1 v}$$

$$\Rightarrow \quad \frac{1}{2}v^2\left[1+\frac{1}{r^2M^2}-\frac{2}{rM^2}\right]-v^2+\frac{v^2}{rM^2}$$

$$= \frac{q^2}{\rho_1 v}$$

$$\Rightarrow \quad \frac{1}{2}v^2 + \frac{v^2}{2r^2M^4} - \cancel{\frac{v^2}{rM^2}} = v^2 - \cancel{\frac{v^2}{rM^2}}$$

$$- \frac{q_2}{\rho_1 v}$$

$$\Rightarrow \quad \frac{v^2}{2r^2M^4} - \frac{v^2}{2} = \frac{q_2}{\rho_1 v}$$

$$\text{OR,} \quad \frac{q_2}{\rho_1} = \frac{1}{2}v^3\left[\frac{1}{r^2M^4}-1\right]$$

$$\frac{R^{k+3}\, t^{\lambda-3}\, F(y_0)}{A R^{\omega}} = \frac{1}{2}\times\frac{b^3}{a^3}\frac{R^3}{t^3}\left[\frac{1}{r^2M^4}-1\right]$$

$$k=\omega, \quad b=2, \quad a=-(4+\omega), \quad \lambda=0$$

$$\frac{R^{\omega+3}F(y_0)}{t^3 R^{\omega} A} = \frac{4}{(4+\omega)^2}\frac{R^3}{t^3}\left[\frac{1}{r^2M^4}-1\right]$$

$$\boxed{\frac{F_{(40)}}{A} = \frac{4}{(4+\omega)^2}\left[\frac{1}{r^2 m^2 4} - 1\right]}$$

and

$$P_2 - P_1 = m_s v_2$$

Substitute the value of $m_s = \rho_1 v$

$$P_2 - P_1 = \rho_1 v u_2$$

$$P_2 = \rho_1 v u_2 + P_1$$

$$P_2 = \rho v . v \quad \left(1 - \frac{1}{rm^2}\right) + \frac{v^2 \rho_1}{m^2 \gamma}$$

$$P_2 = \rho_1 v^2 \quad \left(1 - \frac{1}{rm^2} + \frac{1}{rm^2}\right)$$

$$P_2 = \rho_1 v^2$$

$$R^{K+2} t^{\lambda-2} P_{(40)} = A R^{\omega} \left(-\frac{b}{a} \frac{R}{t}\right)$$

$$R^{K+2} t^{\lambda-2} P_{(40)} = -A \frac{R^{\omega+1}}{t} \frac{b}{a}$$

Substitute the value of k and λ

$$R^{\omega+2} t^{-2} P_{(40)} = -\frac{A}{t} R^{\omega+1} \frac{b}{a}$$

$$R \, P_{(40)} = \frac{A t^2}{(4+\omega)} V$$

$$\frac{P_{(40)}}{A} = \frac{t}{R} \frac{2}{(4+\omega)} V$$

$$\boxed{\frac{P_{(40)}}{A} = \frac{4}{(4+\omega)}}$$

For numerical calculation the flow any field variable have been taken in the following non-dimensional form

$$\frac{u}{u_r} = \frac{\frac{r}{t}\, v(\eta)}{R/t\; v(\eta_0)} = \frac{r}{R}\;\frac{v(\eta)}{v(\eta_0)}$$

but $\eta = r^a t^b$ and $\eta_0 = R^a t^b$

$$\frac{\eta}{\eta_0} = \left(\frac{r}{R}\right)^a$$

or,

$$\frac{r}{R} = \left(\frac{\eta}{\eta_0}\right)^{1/a}$$

Substitute in equation

$$\frac{u}{u_r} = \left(\frac{\eta}{\eta_0}\right)^{1/a} \frac{v(\eta)}{v(\eta_0)}$$

but $a = -(4+\omega)$

$$\frac{u}{u_r} = \left(\frac{\eta}{\eta_0}\right)^{\frac{1}{(4+\omega)}} \frac{v(\eta)}{v(\eta_0)}$$

Similarly

$$\frac{f}{f_2} = \frac{r^{k} t^{1} \Omega(y)}{R^{k} t^{1} \Omega(y_0)} = \left(\frac{r}{R}\right)^{k} \frac{\Omega(y)}{\Omega(y_0)}$$

$$= \left(\frac{y}{y_0}\right)^{k/a} \left(\frac{\Omega(y)/A}{\Omega(y_0)/A}\right)$$

Where $k = \omega$

$$\frac{f}{f_2} = \left(\frac{y}{y_0}\right)^{\frac{\omega}{(4+\omega)}} \left(\frac{\Omega(y)/A}{\Omega(y_0)/A}\right)$$

and

$$\frac{p}{p_2} = \frac{r^{k+2} t^{1-2} P(y)}{R^{k+2} t^{1-2} P(y_0)}$$

$$= \left(\frac{r}{R}\right)^{k+2} \frac{P(y)}{P(y_0)}$$

$$= \left(\frac{r}{R}\right)^{k+2} \frac{P(y)/A}{P(y_0)/A}$$

$$\frac{p}{p_2} = \left(\frac{y}{y_0}\right)^{\frac{k+2}{a}} \frac{P_{(y)}/A}{P_{(y_0)}/A}$$

$$\frac{p}{p_2} = \left(\frac{y}{y_0}\right)^{\frac{\omega+2}{(4+\omega)}} \frac{P_{(y)}/A}{P_{(y_0)}/A}$$

and,

$$\frac{q}{q_2} = \frac{r^{k+3}\, t^{\lambda-3}\, F_{(y)}}{R^{k+3}\, t^{\lambda-3}\, F_{(y_0)}}$$

$$\frac{q}{q_2} = \left(\frac{r}{R}\right)^{k+3} \frac{F_{(y)}/A}{F_{(y_0)}/A}$$

$$\frac{q}{q_2} = \left(\frac{y}{y_0}\right)^{\frac{k+3}{a}} \frac{F_{(y)}/A}{F_{(y_0)}/A}$$

$$\frac{q}{q_2} = \left(\frac{y}{y_0}\right)^{\frac{k+3}{(4+\omega)}} \frac{F_{(y)}/A}{F_{(y_0)}/A}$$

For numerical calculation

$y = y - 1$

CONCLUSION

The differential equations (A) to (D) are numerically solved by Runge-Kutta method. The numerical results for a certain choice of parameters are reproduced in the form of table we conclude that the field parameters decreases in the radiative shock front. The radiative velocity (u), density (ρ), Pressure (p) and heat flux (q) all are decreas in first and second set but shock front is increases in table first and second.

The table of certain choice of parameters with reproduce in the form of tables.

First set

$\gamma = \frac{4}{3}, M^2 = 20, N = 10, w = -1.5, \alpha = \frac{1}{3}$

η	u/u_2	ρ/ρ_2	p/p_2	q/q_2
1.000 00	1.0000000	1.0000000	1.0000000	1.0000000
1.100 00	0.9735775	0.8409035	0.7628222	1.0454151
1.200 00	0.9530111	0.7323471	0.5951056	1.0873912
1.300 00	0.9369498	0.6567086	0.4703231	1.1243515
1.400 00	0.9242489	0.6043024	0.3731744	1.1555710
1.500 00	0.9138566	0.5706075	0.2944013	1.1806336
1.600 00	0.9044285	0.5564229	0.2282242	1.1990562
1.700 00	0.8916298	0.5749437	0.1712666	1.2098198
1.800 00	0.8741511	0.7064606 36	0.1233300	1.2102131
1.900 00	0.8198924	2.9936543	0.0742085	1.1910570
1.910 00	0.8036516	33.050653	0.0413089	1.1848515

TABLE II

Second set

$\gamma = \frac{5}{3}, M^2 = 20, N = 100, w = -1.75, \alpha = \frac{3}{7}$

η	u/u_2	ρ/ρ_2	p/p_2	q/q_2
1.000 00	1.0000000	1.0000000	1.0000000	1.0000000
1.100 00	0.9611185	0.8255005	0.8181402	0.9282533
1.200 00	0.9280842	0.7043508	0.6924694	0.8824891
1.300 00	0.8997621	0.6169417	0.6020795	0.8529418
1.400 00	0.8752778	0.5518644	0.5349040	0.8339617
1.500 00	0.8539[illegible]3	0.5021333	0.4836044	0.8221112
1.600 00	0.8352256	0.4632864	0.4435191	0.8152099
1.700 00	0.8186828	0.4323696	0.4115770	0.8118257
1.800 00	0.8039696	0.4073669	0.3856908	0.8109901
1.900 00	0.7908040	0.3868654	0.3644028	0.8120310
2.000 00	0.7789563	0.3698514	0.3466707	0.8144719
2.100 00	0.7682379	0.3555831	0.3317335	0.8179682
2.200 00	0.7584934	0.3435072	0.3190253	0.8222664
2.300 00	0.7495935	0.3332043	0.3081179	0.8271774
2.400 00	0.7414304	0.3243518	0.2986825	0.8325582
2.500 00	0.7339134	0.3166982	0.2904632	0.8382994
2.600 00	0.7269657	0.3100448	0.2832582	0.8443164

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13. LATTER	(1955)	:	Journal of Applied Physics 26181, 954
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18. TAYLOR	(1950)	:	Proc. Roy. Soc. (London() A201,159
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SPHERICAL SHOCK WAVES IN VISCOUS MAGNETOGASDYNAMICS

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Abstract. The point-source, spherical magnetogasdynamics shock wave moving into a constant density γ-law gas is considered in the limit of infinite shock strength, from the point of view of the Richtmyer–Von Neumann viscosity technique. Numerical solutions of this problem has been obtained in viscous and non-viscous regions. A similarity solution of this problem is shown to exist. We have shown that field variables change rapidly when the magnetic field is imposed in both the viscous and the non-viscous regions.

1. Introduction

The existence of shock waves in gasdynamic flow field introduces free-boundary discontinuities into physical parameter of the system. Such discontinuities cause considerable analytic as well as numerical complications in the treatment of gasdynamic problems. A method for avoiding such difficulties, particularly for the numerical calculations, has been developed by Richtmyer and Von Neumann (1950). They observed that the addition of a particular viscosity like term into gasdynamic equations could lead to continuous shock flow field in which the finite thickness of discontinuities at the shock wave was removed and replaced by a region in which the physical parameters changed rapidly, but smoothly. Following this method, Lax (1954), Latter (1955), Brode (1954), Colgate and Johnson (1960), and Christy (1964) solved various shock problems. Sachdev and Prasad (1966) used the method of artificial heat conduction to smear out the shock in ordinary gasdynamics.

In the present work, we use the artificial mechanism of viscosity in the presence of magnetic field to smear out discontinuities of the physical parameters from the flow field. In order to give a meaning to the otherwise physically unrealisable magnetic field with the spherical symmetry, the magnetic field is replaced by an idealized field such that lines of forces lie on a hemisphere whose centre is the point of explosion (cf. Summers, 1975). We have used the Runge–Kutta method to obtain numerical solutions in viscous and inviscid regions. The numerical calculations have been done on a DEC-system 1090 computer installed at I.T.T. Kanpur by RKGS programma.

2. Formulation of the Problem

The equations of motion of a fluid having infinite electrical conductivity when expressed in spherically-symmetric Eulerian form with artificial viscosity term as suggested by

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Richtmyer and Von Neumann are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial}{\partial r}(p+q) + \frac{H}{\rho}\frac{\partial H}{\partial r} + \frac{H^2}{\rho r} = 0\,, \tag{1}$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right) = 0\,, \tag{2}$$

$$\frac{\partial H}{\partial t} + u\frac{\partial H}{\partial r} + H\frac{\partial u}{\partial r} + \frac{Hu}{r} = 0\,, \tag{3}$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r} - \frac{\{\gamma p + (\gamma - 1)q\}}{\rho}\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) = 0\,, \tag{4}$$

where p, u, ρ, H, and q are pressure, velocity, density, magnetic field, and artificial viscosity, respectively. Magnetic permeability of the medium is taken to be unity throughout the problem.

The caloric equation of state of the medium is assumed as

$$\varepsilon = \frac{p}{\rho(\gamma - 1)}\,, \tag{5}$$

where ε is the internal energy per unit mass. The adiabatic equation of state is assumed as,

$$\frac{p}{\rho\gamma} = \sigma(s)\,, \tag{6}$$

where $\sigma(s)$ is a function of entropy s only. Although the dependence of q and p, ρ and u which prohibits discontinuities in the physical parameters is not uniquely prescribed, the form of q in the present discussion consistent with the requirement of Richtmyer and Von Neumann is

$$q = \frac{1}{2}K^2\rho r^2\,\frac{\partial u}{\partial r}\left(\left|\frac{\partial u}{\partial r}\right| - \frac{\partial u}{\partial r}\right). \tag{7}$$

The form of q chosen in (7) has been made from the point of view of admitting similarity solutions, since all forms of q may not fulfil this condition.

We assume the existence of a similarity solution of the form

$$p(r,t) = \rho_0 R^{-\beta} f(x)\,, \qquad \rho(r,t) = \rho_0 \psi(x)\,, \tag{8}$$
$$\mu(r,t) = R^{-\alpha}\quad \phi(x)\,, \qquad H(r,t) = \sqrt{\rho_0 R^{-\beta}}\,\eta(x)\,,$$

where $x = r/R$, R being a function of time only, ρ_0 is an orbitrary constant having dimension of density, and α, β are constants. To simplify the subsequent calculations,

it is assumed that $\alpha = \frac{1}{2}\beta$. We change the independent variables from (r, t) to (x, ζ) with the aid of the following relations and later put $\zeta = t$: we find that

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t} - \frac{x\dot{R}}{R}\frac{\partial}{\partial x}, \qquad \frac{\partial}{\partial r} = \frac{1}{R}\frac{\partial}{\partial x},$$

where

$$\dot{R} = \frac{dR}{dt}.$$

Equations (1) to (4) then take the forms

$$-A(\alpha\phi + x\phi') + \phi\phi' + \frac{f' + g' + \eta\eta'}{\psi} + \frac{\eta^2}{\psi x} = 0, \tag{9}$$

$$\psi'(\phi - Ax) + \psi\phi' + \frac{2\psi\phi}{x} = 0, \tag{10}$$

$$\eta'(\phi - Ax) + \eta\left(\phi' - \alpha A + \frac{\phi}{x}\right) = 0, \tag{11}$$

$$-2A\alpha f + (\phi - Ax)f' - \frac{\gamma f + (\gamma - 1)}{\psi}(\phi - Ax) = 0, \tag{12}$$

where a prime denotes the differentiation with respect to x, and the quantity g is related to q by the relation

$$\frac{q}{\rho_0} = \frac{K^2}{2R^{2\alpha}}\psi x^2\phi'(|\phi'| - \phi) = \frac{g(x)}{R^{2\alpha}}. \tag{13}$$

It is assumed that

$$R^{\alpha}\dot{R} = A \quad \text{(a constant)}.$$

If E be the total energy, then

$$E = 4\pi\int_0^{\infty} r^2\left(\rho\frac{u^2}{2} + \frac{p}{\gamma - 1} + \frac{H^2}{2}\right)dr, \tag{14}$$

which becomes in the terms of present similarity variable as

$$E = 4\pi R^{3-2\alpha}\rho_0\int_0^{\infty}\left(\frac{\psi\phi^2}{2} + \frac{f}{\gamma - 1} + \frac{\eta^2}{2}\right)x^2\,dx. \tag{15}$$

Since R is a function of time only, the energy E will be independent of time if $\alpha = \frac{3}{2}$.

We now introduce new dimensionless quantities

$$F = \frac{f}{A}\,, \qquad G = \frac{g}{A}\,, \qquad P = \frac{\phi}{A}\,, \qquad N = \frac{\eta}{A}\,. \tag{16}$$

Transforming Equations (9) to (13) and (15) with the aid of (16), we get

$$-\left(\tfrac{3}{2}P' + xP'\right) + PP' + \frac{F' + G' + NN'}{\psi} + \frac{N^2}{\psi x} = 0\,, \tag{17}$$

$$\psi'(P - x) + \psi P' + 2\psi\,\frac{P}{x} = 0\,, \tag{18}$$

$$N'(P - x) + N\left(P' + \frac{P}{x} - \frac{3}{2}\right) = 0\,, \tag{19}$$

$$-3F + (P - x)F' - \frac{\gamma F + (\gamma - 1)G}{\psi}\,(P - x)\psi' = 0\,, \tag{20}$$

$$G = \frac{K^2}{2}\,x^2 P'\left(|P'| - P'\right), \tag{21}$$

and

$$E = 4\pi\rho_0 A^2 \int_0^\infty \left(\frac{\psi P^2}{2} + \frac{F}{\gamma - 1} + \frac{N^2}{2}\right) x^2\,\mathrm{d}x\,. \tag{22}$$

Taylor (1950) has shown the existence of a solution of equation of the type (17) to (21) for a diverging flow from a point source with a shock discontinuity at $x = 1$, since for such a diverging flow $P' > 0$ and so $G = 0$.

The treatment of Equations (17) to (20) and Equation (21) will be divided separately for the region where the viscosity is absent and the region where it is present. First, the region without viscosity will be treated and, thereafter, the region ith viscosity will be explored. They would later be matched through numerical integration.

3. Solution of the Equations (17)–(21)

3.1. Inviscid Flow

From the boundary condition for a diverging flow field it follows that, at $r = R$, the gradient of velocity is positive and, therefore, the viscosity term is zero – i.e., $G = 0$. The equations defining the flow and the field equation reduce to

$$-\left(\tfrac{3}{2}P + xP'\right) + PP' + \frac{F' + NN'}{\psi} + \frac{N^2}{\psi x} = 0\,, \tag{23}$$

$$(P-x)\psi' + \psi P' + 2\psi \frac{P}{x} = 0\,, \tag{24}$$

$$N'(P-x) + N\left(P' + \frac{P}{x} - \frac{3}{2}\right) = 0\,, \tag{25}$$

$$-3F + (P-x)F' - \gamma F(P-x)\frac{\psi'}{\psi} = 0\,. \tag{26}$$

We use the boundary condition to obtain the numerical solution of the above equations

3.2. VISCOUS FLOW

In order to introduce viscosity, it is essential to have discontinuities in the distribution of flow variables. We may arbitrarily locate the position of these discontinuities at $r = R$; so that the problem now remains to obtain the solution in the viscous region $x \geq 1$, for which $P' \leq 0$.

Equation (21) may be written as

$$P' = -\frac{1}{Kx}\left(\frac{G}{\psi}\right)^{1/2}; \tag{27}$$

at the transition point $x = 1$, $G = 0$, and so by Equation (27) $P' = 0$. This condition determines the magnitude of jump in the slope P' in the transition from $x = 1$ onwards.

4. Results and Discussion

To illustrate the behaviour of the present similarity problem, the solution of Equations (23)–(26) are obtained for $\gamma = 1.4$ in the region without viscosity $(0 \leq x \leq 1)$ and Equations (17) to (20) and (27) are solved numerically for the viscous region $(x \geq 1)$. We have used the following boundary conditions

$$P(1) = (1-\delta)\,, \qquad F(1) = (1-\delta)\,, \qquad \psi(1) = \frac{1}{\delta}\,, \qquad N(1) = \frac{M_A{}^{1}}{\delta}\,,$$

where δ is the ratio of densities just ahead and just behind the shock front, i.e., $\rho_0/\rho_s = \delta$ and Alfvén's Mach number $M_A = V\sqrt{\rho_0}/H$. The ratio of densities – i.e., ρ_s/ρ_0 for $\gamma = 1.4$ is 6 for the Taylor problem; but for $K = 0.0349$ it is 4.054 and for $K = 0.349$ it is 1.137 (see Latter, 1955). We have calculated our problem for these values in the presence of the magnetic field.

We conclude that the field parameters change rapidly in the inviscid region as well as in viscous region when the the magnetic field is imposed. The variation of velocity, pressure, density, magnetic field, and viscosity for both the viscous and non-viscous region has been illustrated through Figure 1 to Figure 9. We also infer that in viscous region, the field parameters attain a maximum value at the shock front when the artificial

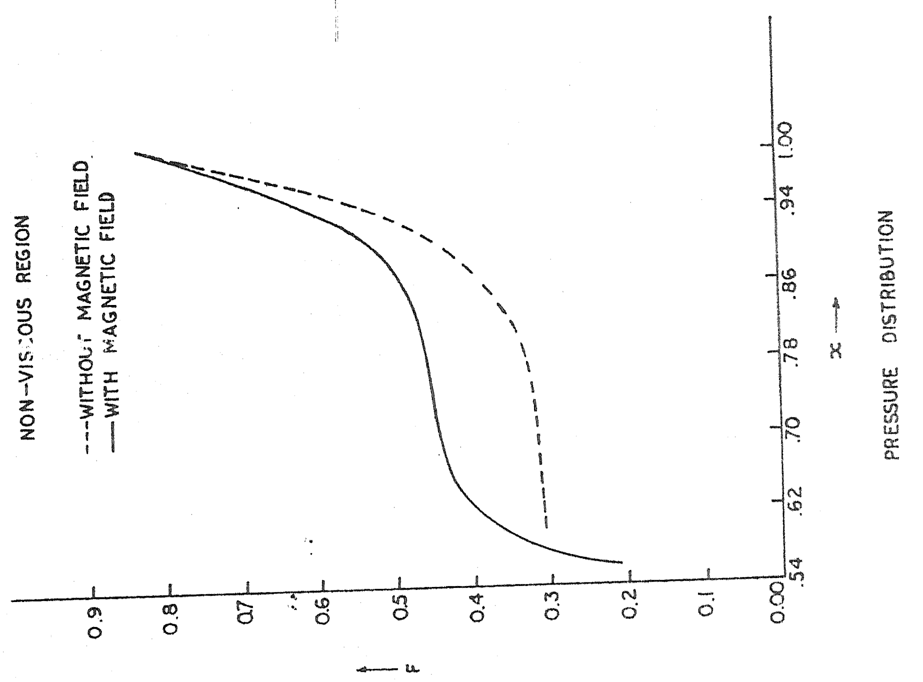

Fig. 2. Pressure distribution.

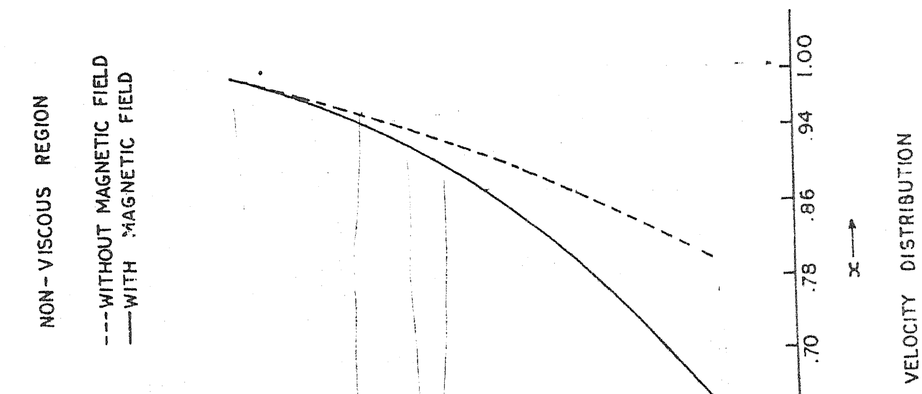

Fig. 1. Velocity distribution.

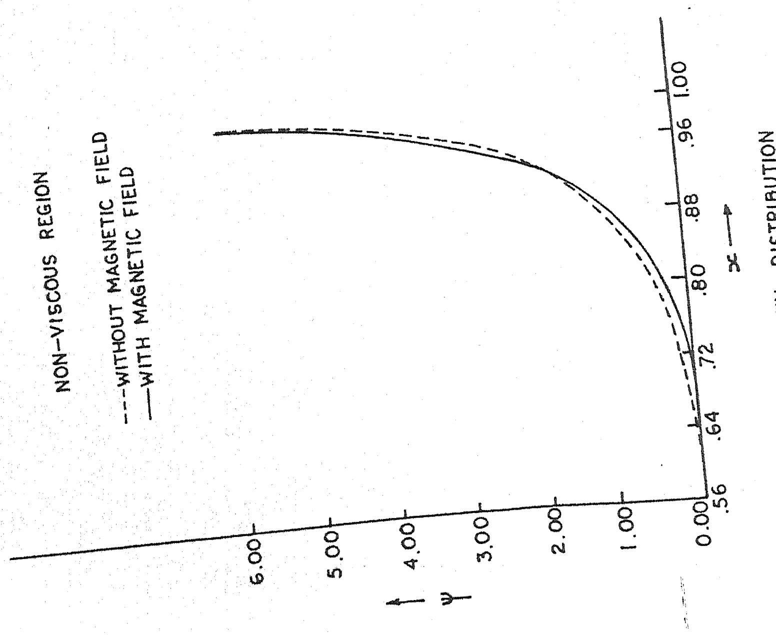

Fig. 3. Density distribution.

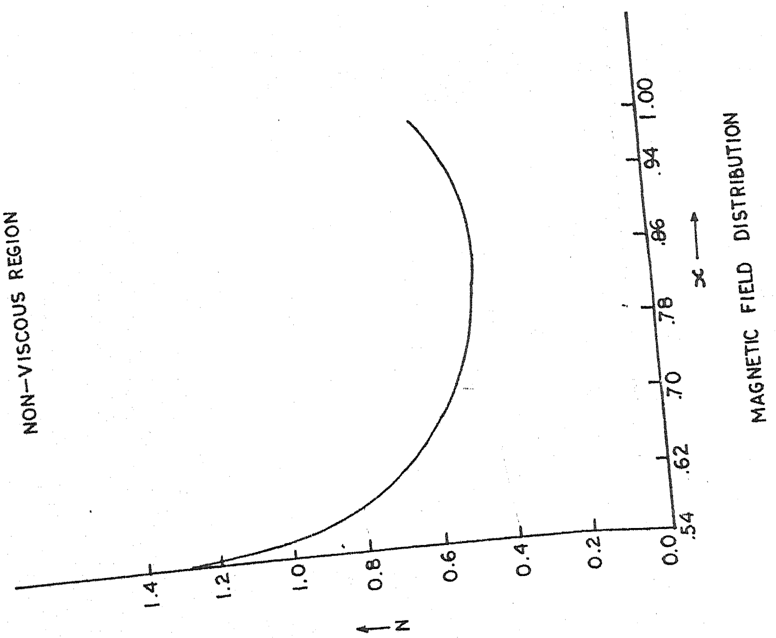

Fig. 4. Magnetic field distribution.

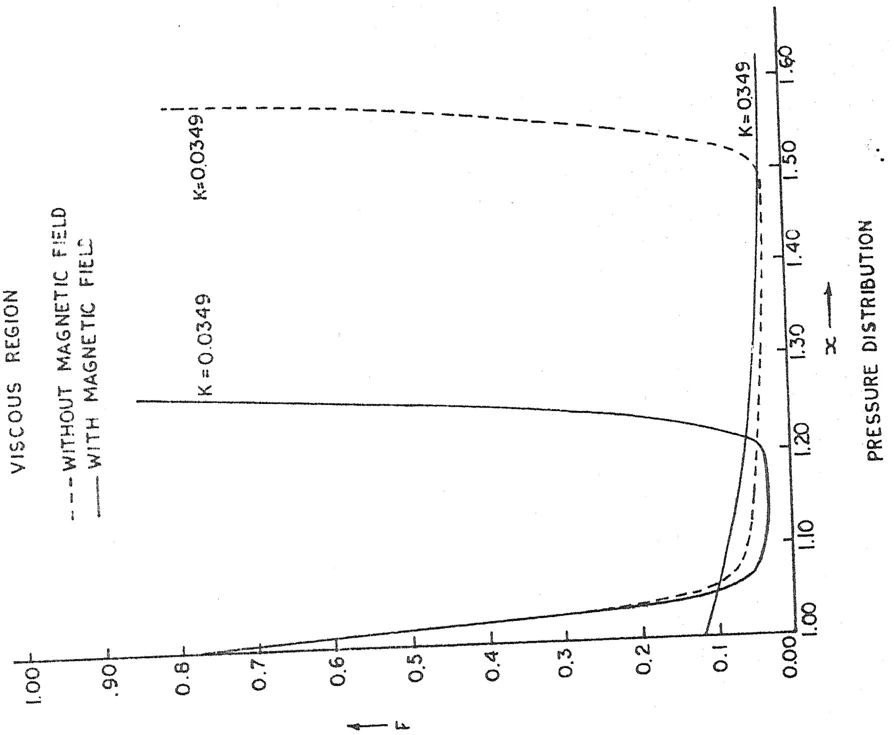

Fig. 6. Pressure distribution.

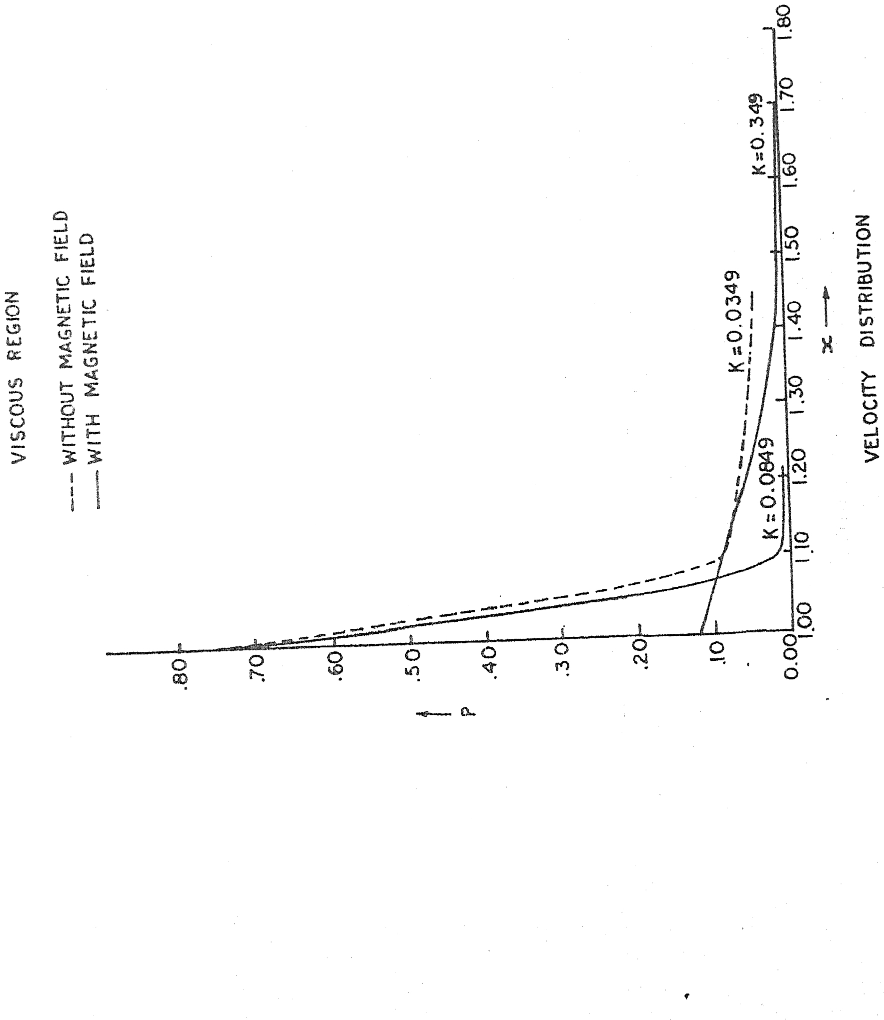

Fig. 5. Velocity distribution.

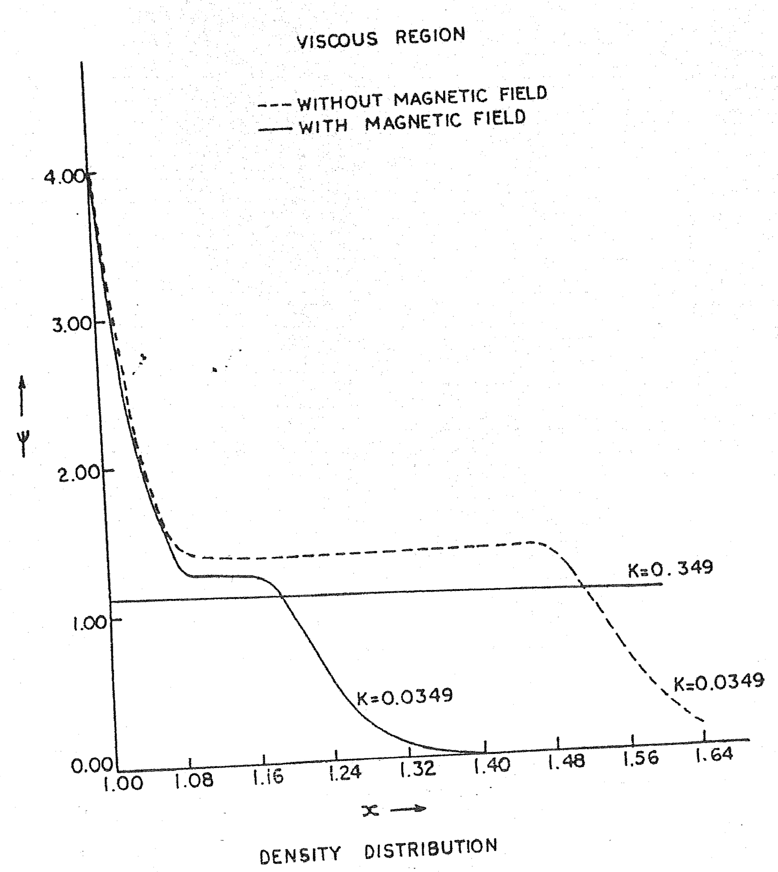

Fig. 7. Density distribution.

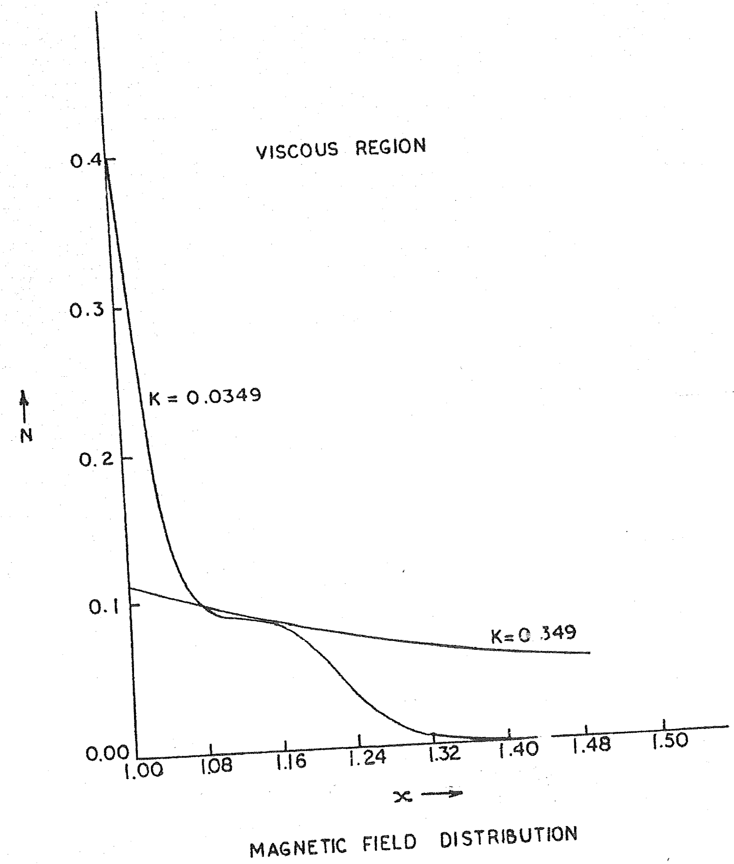

Fig. 8. Magnetic field distribution.

SELF-SIMILAR CYLINDRICAL SHOCK WAVES WITH RADIATION HEAT FLUX

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Abstract. Similarity solutions describing the flow of a perfect gas behind cylindrical shock waves with radiation heat flux are investigated. The total energy of the expanding wave has been supposed to remain constant. The solution, however, is only applicable to a gaseous medium where the undisturbed pressure falls as the inverse square of the distance from the line of explosion.

1. Introduction

The problem of propagation of shock waves in a non-homogeneous medium is of great interest in exploring the effect of explosion in the stars and atmosphere of the Earth.

The solution for cylindrically symmetric flow has been obtained numerically by Lin (1954). Ray (1957) has discussed the problems of point and line explosion and found an exact analytic solution. Analytic solutions in the three cases of plane, cylindrical and spherical flow have been noted by Sakurai (1955). Rogers (1958) has also studied the similarity solutions for these three cases in uniform atmosphere.

In the present paper the problem of explosion along a line in a gas cloud has been discussed. Similarity solutions have been developed describing the propagation of a cylindrical shock in non-uniform atmosphere taking counter gas pressure and radiation heat flux into account. The radiation pressure and radiation energy have been ignored. The gas in the undisturbed field is assumed to be at rest. We also have assumed the gas to be grey and opaque and the shock to be transparent and isothermal. The total energy of the explosion is constant.

2. Self-Similar Formulation

The cylindrical polar coordinates, when r is the radial distance from the line of explosion, have been used here. The equation of conservation of mass, momentum and energy behind the wave are

$$\frac{D\rho}{Dt} + \frac{\rho}{r}\frac{\partial}{\partial r}(ru) = 0 , \qquad \frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + \rho\frac{\partial u}{\partial r} + j\left(\frac{\rho u}{r}\right) = 0 \tag{1}$$

$$\frac{Du}{Dt} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0 , \qquad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{h}{\rho}\frac{\partial h}{\partial r} + \frac{\partial h^2}{\rho r} = 0 \tag{2}$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} + \nu\frac{hu}{r} = 0$$

conditions, following Singh and Vishwakarma (1983), are compatible when

$$k = \omega\,, \quad \lambda = 0\,, \quad a = -(4+\omega)\,, \quad b = 2\,,$$

$$n = -2\,, \quad \beta = -\tfrac{5}{2} \quad \text{and} \quad \alpha = \frac{\omega+1}{\omega}\,. \tag{20}$$

$n = \frac{\omega+2}{2}$

Hence, the pressure distribution becomes

$$p_1 = BR^{-2}\,. \tag{21}$$

Equations (1)–(3) and ([illegible]) are now transformed into the forms

$$\frac{\Omega'(\eta)/A}{\Omega(\eta)/A} = \frac{\eta(4+\omega)v'(\eta) - (2+\omega)v(\eta)}{\eta[2-(4+\omega)v(\eta)]}\,, \tag{22}$$

$$\frac{P'(\eta)/A}{P(\eta)/A} = \frac{\Omega(\eta)/A}{P(\eta)/A}\left[\frac{\eta v'(\eta)\,\{2-(4+\omega)v(\eta)\} + v(\eta)\,(v(\eta)-1)}{\eta(4+\omega)}\right] + \\ + \frac{2+\omega}{\eta(4+\omega)}\,, \tag{23}$$

$$\frac{F'(\eta)/A}{F(\eta)/A} = \frac{P'(\eta)/A}{F(\eta)/A}\left[\frac{2-(4+\omega)v(\eta)}{(\gamma-1)(4+\omega)}\right] + \\ + \frac{P(\eta)/A}{F(\eta)/A}\left[\frac{(\omega+2\gamma+2)v(\eta) - \gamma(4+\omega)\eta v'(\eta) - 2}{\eta(\gamma-1)\,(4+\omega)}\right] + \frac{1}{\eta}\,, \tag{24}$$

$$\frac{F(\eta)/A}{P(\eta)/A} = -N\,\frac{(P(\eta)/A)^{1/2}}{(\Omega(\eta)/A)^{(3/2-\alpha)}}\left[2-(4+\omega)\eta\left\{\frac{P'(\eta)/A}{P(\eta)/A} - \frac{\Omega'(\eta)/A}{\Omega(\eta)/A}\right\}\right], \tag{25}$$

where

$$N = \frac{4\sigma C\mu_0}{3\Gamma^{3/2}A^{1-\alpha}} = \text{a non-dimensional radiation parameter}\,, \tag{26}$$

$$\frac{1}{N}\,\frac{(F(\eta)/A)\,(\Omega(\eta)/A)^{(1/2-\alpha)}}{(P(\eta)/A)^{1/2}}\,[2-(4+\omega)v(\eta)] - 2\,\frac{P(\eta)/A}{\Omega(\eta)/A} \times \\ \times v'(\eta) = \frac{[(4+\omega)v(\eta)+\omega] - [2-(4+\omega)v(\eta)]\,[v(\eta)\,(v(\eta)-1)]}{\eta\left[\{2-(4+\omega)v(\eta)\}^2 - (4+\omega)^2\,\dfrac{P(\eta)/A}{\Omega(\eta)/A}\right]}\,. \tag{27}$$

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$$\frac{F'(\eta)/A}{F(\eta)/A} = \frac{P'(\eta)/A}{F(\eta)/A}\left[\frac{2-(4+\omega)v(\eta)}{(\gamma-1)(4+\omega)}\right] + \\ + \frac{P(\eta)/A}{F(\eta)/A}\left[\frac{(\omega+2\gamma+2)v(\eta) - \gamma(4+\omega)\eta v'(\eta) - 2}{\eta(\gamma-1)\,(4+\omega)}\right] + \frac{1}{\eta}\,, \tag{24}$$

$$\frac{F(\eta)/A}{P(\eta)/A} = -N\,\frac{(P(\eta)/A)^{1/2}}{(\Omega(\eta)/A)^{(3/2-\alpha)}}\left[2-(4+\omega)\eta\left\{\frac{P'(\eta)/A}{P(\eta)/A} - \frac{\Omega'(\eta)/A}{\Omega(\eta)/A}\right\}\right], \tag{25}$$

where

$$N = \frac{4\sigma C\mu_0}{3\Gamma^{3/2}A^{1-\alpha}} = \text{a non-dimensional radiation parameter}\,, \tag{26}$$

$$\frac{1}{N}\,\frac{(F(\eta)/A)\,(\Omega(\eta)/A)^{(1/2-\alpha)}}{(P(\eta)/A)^{1/2}}\,[2-(4+\omega)v(\eta)] - 2\,\frac{P(\eta)/A}{\Omega(\eta)/A} \times \\ \times v(\eta)\,\frac{[(4+\omega)v(\eta)+\omega] - [2-(4+\omega)v(\eta)]\,[v(\eta)\,(v(\eta)-1)]}{\eta\left[\{2-(4+\omega)v(\eta)\}^2 - (4+\omega)^2\,\dfrac{P(\eta)/A}{\Omega(\eta)/A}\right]}\,. \tag{27}$$

$u_2 = \frac{1}{2}\left[1 - \frac{1}{\gamma M^2}\right]V, \quad \rho_2 = \rho_1 \gamma M^2, \quad p_2 = \rho_1 V^2,$

$q_2 = -\frac{1}{2}\rho_1 V^3\left[1 - \frac{1}{\gamma^2 M^4}\right]$

The approximate shock conditions are

$$v(\eta_0) = \frac{2}{(4 + \omega)}\left[1 - \frac{1}{\gamma M^2}\right], \tag{28}$$

$$\frac{\Omega(\eta_0)}{A} = \gamma M^2 , \tag{29}$$

$$\frac{P(\eta_0)}{A} = \frac{4}{(4 + \omega)^2} , \tag{30}$$

$$\frac{F(\eta_0)}{A} = \frac{1}{2}\left(\frac{2}{4 + \omega}\right)^3\left[\frac{1}{\gamma^2 M^4} - 1\right]; \tag{31}$$

which are the initial values for our numerical calculation, where we assume that $\eta_0 = 1$.

3. Results

Differential equations are numerically solved by the Runge–Kutta technique and the solutions are presented in a convenient form as

$$\frac{u}{u_2} = \left(\frac{\eta_0}{\eta}\right)^{1/(4+\omega)} \frac{v(\eta)}{v(\eta_0)} , \tag{32}$$

$$\frac{\rho}{\rho_2} = \left(\frac{\eta_0}{\eta}\right)^{\omega/(4+\omega)} \frac{\Omega(\eta)/A}{\Omega(\eta_0)/A} , \tag{33}$$

$$\frac{p}{p_2} = \left(\frac{\eta_0}{\eta}\right)^{(2+\omega)/(4+\omega)} \frac{P(\eta)/A}{P(\eta_0)/A} , \tag{34}$$

and

$$\frac{q}{q_2} = \left(\frac{\eta_0}{\eta}\right)^{(3+\omega)/(4+\omega)} \frac{F(\eta)/A}{F(\eta_0)/A} . \tag{35}$$

The numerical results for a certain choice of parameters are reproduced in the form of tables. Nature of the field variables may be seen through these Tables I and II. We calculate our results for the following two sets of parameters:

(i) $\gamma = \frac{4}{3}$, $M^2 = 20$, $N = 10$, $\omega = -1.5$, $\alpha = \frac{1}{3}$,

(ii) $\gamma = \frac{5}{3}$, $M^2 = 20$, $N = 100$, $\omega = -1.75$, $\alpha = \frac{3}{7}$.

Numerical integration was carried out on the DEC-System 1090 computer installed at the IIT Kanpur, using well-known RKGS programme. The second set of parameters are more effective on the flow variables rather than the first set. The radiation parameter N affects the variation of density, pressure, and radiation heat flux as its value increases.

TABLE I

First set

η	u/u_2	ρ/ρ_2	p/p_2	q/q_2
	$\gamma = \frac{4}{3}$, $M^2 = 20$, $N = 10$, $w = -1.5$, $\alpha = \frac{1}{3}$			
1.000 00	1.000 000 0	1.000 000 0	1.000 000 0	1.000 000 0
1.100 00	0.973 577 5	0.840 903 5	0.762 822 2	1.045 415 1
1.200 00	0.953 011 1	0.732 347 1	0.595 105 6	1.087 391 2
1.300 00	0.936 949 8	0.656 708 6	0.470 323 1	1.124 351 5
1.400 00	0.924 248 9	0.604 302 4	0.373 174 4	1.155 571 0
1.500 00	0.913 836 6	0.570 607 5	0.294 401 3	1.180 633 6
1.600 00	0.904 428 5	0.556 422 9	0.228 224 2	1.199 056 2
1.700 00	0.893 629 8	0.574 943 7	0.171 266 6	1.209 819 8
1.800 00	0.874 151 1	0.706 466 36	0.123 330 0	1.210 213 1
1.900 00	0.819 892 4	2.993 654 3	0.074 208 5	1.191 057 0
1.910 00	0.803 651 6	33.050 653	0.041 308 9	1.184 851 5

TABLE II

Second set

η	u/u_2	ρ/ρ_2	p/p_2	q/q_2
	$\gamma = \frac{5}{3}$, $M^2 = 20$, $N = 100$, $w = -1.75$, $\alpha = \frac{3}{7}$			
1.000 00	1.000 000 0	1.000 000 0	1.000 000 0	1.000 000 0
1.100 00	0.[illegible] 118 5	0.825 500 5	0.818 140 2	0.928 253 3
1.200 00	0.928 084 2	0.704 350 8	0.692 469 4	0.882 489 1
1.300 00	0.899 762 1	0.616 941 7	0.602 079 5	0.852 941 8
1.400 00	0.875 277 8	0.551 864 4	0.534 904 0	0.833 961 7
1.500 00	0.853 946 3	0.502 133 3	0.483 604 4	0.822 111 2
[illegible] 600 [illegible]	0.835 2[illegible]5 6	0.463 286 4	0.4[illegible]3 [illegible]9 1	0.81[illegible] 209 [illegible]
1.700 00	0.818 682 8	0.432 369 6	0.411 [illegible]77 0	0.81[illegible] 825 7
1.800 00	0.803 969 6	0.407 366 9	0.385 690 8	0.810 990 1
1.900 00	0.790 804 0	0.386 865 4	0.364 402 8	0.812 031 0
2.000 00	0.778 956 3	0.369 851 4	0.346 670 7	0.814 471 9
2.100 00	0.768 237 9	0.355 583 1	0.331 733 5	0.817 968 2
2.200 00	0.758 493 4	0.343 507 2	0.319 025 3	0.822 266 4
2.300 00	0.749 593 5	0.333 204 3	0.308 117 9	0.827 177 4
2.400 00	0.741 430 4	0.324 351 8	0.298 682 5	0.832 558 2
2.500 00	0.733 913 4	0.316 698 2	0.290 463 2	0.838 299 4
2.600 00	0.726 965 7	0.310 044 8	0.283 258 2	0.844 316 4

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